Phys 332
Electricity and Magnetism II
Prof. Fulvio Melia
Computer Projects

There are four possible projects, of which you need to complete one. You would recognize these from Phys 331. So if you did one of those, choose a different one this semester.

Guidelines

1. All text must be typed.

2. All analytical calculations must be typed as well, either using LateX, if you are familiar with it, or word, or any other medium of your choice.

3. It should look like a professional presentation.

4. Also provide a printout of your code. You can choose any language or approach you wish, but the calculation needs to be numerical.

5. We will be dividing the class evenly among the four projects. It is first come first serve. E-mail me your choice and I will pencil you in for your preferred category. In the end, we may have to move people around in order to weight the projects evenly.

6. Projects are due in class on Thursday, April 21.

1: Field due to a Charged Sphere with a tiny hole

Introduction

If we have a spherical shell of radius a and total charge Q, we know that $\vec{E}_{\text{inside}} = 0$, while $\vec{E}_{\text{outside}} = \frac{Q\hat{r}}{4\pi\epsilon_0 r^2}$. Thus, there is a discontinuity in the electric field right at the surface of the sphere due to the surface charge density on the sphere. But what happens if there was a small part of the spherical surface that wasn't charged? That is what you are going to investigate here.

Part I: Analytic calculation of the Field

Your sphere is uniformly charged except for the region where $\theta \leq 1°$ (which has $\sigma = 0$). Imagine that your field point is somewhere on the positive z-axis (so z could be larger or smaller than a). Determine $\vec{E}$ as a function of z.

Part II: Numerical Calculation of the Field

Now write a computer program to do the integral. You should evaluate the field at $z = 0.01na$ where n is an integer. Do this from $n = 0$ to $n = 500$. You will also need to give me a printout of your code.

Part III: Graphing/interpreting the results

On the same graph, graph the *function* you obtained above in Part I and the points you obtained in Part II. Do not connect the points with a line. Verify that both calculations gave the same answer. If the graph is too messy, you may want to subdivide it into several graphs ($z = 0 \to a$, $z = a \to 2a$, etc.) or you may want to try just plotting a fraction of the points you calculated. If things are varying too much, you may need to use the log of your results. You will have to decide how to display your data in a useful way.

Is the change in $\vec{E}$ as you go from $z < a$ to $z > a$ what you would expect? Explain.

You should also graph the "difference" between the field and what it would be if the sphere didn't have the hole (for points with $z > a$). Is that "difference" larger near the surface of the sphere or is it larger farther away? Does your result make sense? Explain.

Part IV: Conclusion

What have you learned from this calculation? Consequences?

2: Motion of a Charged particle in a Dipole Field

Introduction

Imagine that you have a "point dipole" at the origin with $\vec{p} = p_0\hat{z}$. Assume that this dipole is not allowed to move. A positive point charge q is released from rest at a point in the xy plane. What happens to it? That is what you are going to investigate here.

Part I: Just Think....

Using what you know about the structure of the dipole field, explain in words what you think will happen to that charge. Note that you need to turn this in to me **BEFORE** doing any of the rest of this project. I will look over it and discuss it with you. Once that has happened, you can continue with the rest of the project.

Part II: Numerical Calculation of the Motion

Now write a computer program to solve Newton's Laws and determine the trajectory of the charge. Follow it long enough so that you basically know **exactly** what it will do.

Do this a second time in which you let p_0 represent a stronger dipole. So specifically, let $p_0 \to 10p_0$. What change occurs as a result of this? Be specific.

Graph the trajectory. Turn this in to me. I will let you know if it is correct/incorrect. You will also need to give me a printout of your code.

Part III: An alternative method of solving...

Once you have shown me your trajectory, I will give you part three of the assignment. It should be easy and should not take much time. However, if I gave it to you now, it would give away the answer!

Part IV: Conclusion

What have you learned from this calculation? At this time turn everything back in.

3: Field Near a Parallel Plate Capacitor

Introduction

You will be using numerical integration to help you examine the electric field near a parallel plate capacitor. Assume that you have two circular plates. Each has radius a and they are a distance d apart. Assume they are concentric with the z-axis and that the $z = 0$ plane lies exactly halfway between the plates. For Part II and III below, you will have to calculate for the following three scenarios: $\frac{a}{d} = 25$, $\frac{a}{d} = 5$ and $\frac{a}{d} = 1$.

Part I: Setting up the integral

Write out the detailed integral expression for the electric field at a random point (x, y, z). As you can see this is a very messy integral.

Part II: Variation of $\vec{E}$ along the symmetry axis

You need to produce a graph of $\vec{E}$ versus z along the z-axis (both inside and outside the capacitor). To obtain the data for this graph, you can either do the integral analytically or numerically (your choice).

You need to determine if these results make sense. Thus, you should think about what your solutions should look like in various limits. Equivalently, you need to think about what your capacitor plates look like from various positions. Compare your results with whatever you think is appropriate.

You will have to choose what $z's$ to plot. For example, should you plot from $z = -5a$ to $z = +5a$? Is that a big enough range of z? Is it too large a range of z? You may want to change this range for the three different scenarios. Again, you want to be able to compare your results with something logical.

Similarly, you may find that a graph of $\vec{E}$ versus z has too much range. It may be that you need to graph $\log E$ versus z (where E is now a dimensionless electric field) or $\vec{E}$ versus $\log z$ (where z is now a dimensionless coordinate). You need to decide what allows you to display your data best.

Part III: Variation of $\vec{E}$ with ρ

Your now need to produce a graph of $\vec{E}$ versus ρ in the $z = 0$ plane. Since this means you will be examining $\vec{E}$ at points off the symmetry axis, you will have to perform the integral numerically.

You will again need to determine exactly how best to graph this.

You also need to make whatever comparisons you can with other physical situations.

Part IV: Conclusion

What have you learned from this calculation?

4: Relaxation solution of Laplace's equation

You have discussed some of the general properties of solutions to Laplace's Equation. Specifically, you can determine the potential at a particular point by averaging the potential over a sphere centered on that point $\left(\Phi(x_0, y_0, z_0) = \frac{1}{4\pi R^2} \oiint_S \Phi(x,y,z)da \right)$. We are going to use this idea to solve a boundary value problem in a difficult geometry.

Part I: But if we integrate numerically won't there always be gaps in the sphere?

One of the problems with doing integrals numerically is that you will never have the resolution you do analytically. Let's investigate how much of a problem that will be.

Assume you know $\Phi(x,y,z)$. Use that function to approximate $\Phi(x+\delta,y,z)$, $\Phi(x-\delta,y,z)$, $\Phi(x,y+\delta,z)$, $\Phi(x,y-\delta,z)$, $\Phi(x,y,z+\delta)$ and $\Phi(x,y,z-\delta)$ using Taylor expansions accurate to 4th order in δ. How well do you think these six points (which basically sit on the faces of a cube) would approximate a spherical surface?

Determine the average of those six points. You should be able to show that as long as $\Phi(x,y,z)$ satisfies Laplace's Equation then the average of those points is $\Phi(x,y,z)$ to third order in δ (in other words the first error is of order δ^4).

Thus, if you choose six ("cubical") points like that their average potential can be made to equal the potential at the center as accurately as you want *if* you make δ sufficiently small.

Part II: Numerical Solution to a 2D problem - low resolution.

Let's imagine that we have an infinitely long square conductor as shown (see diagram on last page). The inner conductor is maintained at 100 V while the outer conductor is grounded. We want to determine the potential at interior points. We can do this by averaging over the potential at the four nearest points. (Thus, we are doing a 2D version of the general problem referred to in Part I.)

Assume that the outer conductor is thirty "units" long so that thirty-one evenly spaced points can be used to describe it. Similarly, assume that the inner conductor is six "units" long so that seven evenly spaced points can be used to describe it. This is approximately illustrated on the diagram (11 points instead of 31 and 3 instead of 7). You then fill in the rest of the grid with points whose potential you want to calculate.

For example, you can calculate the potential at the (larger) point in the upper right of the diagram by averaging the potentials from the four points around it. Clearly, this will work best when those four points are very close to it.

Thus, you begin by "guessing" the potential at all interior points. Think carefully about this and come up with some scheme. You know the potential drops from 100 V to 0 V and you know that it does this smoothly (since all solutions to Laplace's equation must have this property). Then you calculate the potential at each point by averaging the guesses around it. (You certainly have a choice here. You could use all the "old" values of the potential to calculate the new ones or you could use the new ones as they appear. You may want to try both methods.)

Then you do the averaging again and again and again. Hopefully, the changes in the potential from one step to the next will shrink so that ultimately you have a "relaxed solution". You may want to try letting the code run until the largest change for any point is 0.1% and then again with 0.001% and then again with 0.00001%. Are the results the same? Comparing these graphically will tell you when your criteria is strict enough.

Part III: Numerical Solution to a 2D problem - high resolution.

Repeat the previous part but this time the inner conductor is 30 units long and the outer conductor is 150 units long. Thus, you have five times higher resolution in *each* direction and that should make a significant difference.

If you run the solution for a long time, determine how small the changes are from step to step.

Part IV: Graphing the results.

Make the following graphs:

1: Graph Φ versus distance along a 45° line connecting a corner of the inner conductor to the adjacent corner of the outer conductor. Show the results of the low resolution calculation for the three different convergence criteria described above (0.1%, etc.)

2: Graph Φ versus y along the vertical line from the middle of the inner conductor to the middle of the outer conductor. Show the 0.00001% low resolution result and the high resolution result on the same graph.

3: Graph Φ versus distance along a 45° line connecting a corner of the inner conductor to the adjacent corner of the outer conductor. Show the 0.00001% low resolution result and the high resolution result on the same graph.

Part V: Discussion.

Compare the solutions from low and high resolution.

Do the final numerical values make sense?

Pick a random point somewhere in the grid. Do not pick a point along a line of symmetry. Determine $\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2}$ at that point using the values at the gridpoints to approximate derivatives. How close is that sum to zero? (Make sure you understand what a big difference from zero is here and what a small difference from zero is here.)

Any other comments?