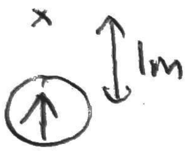
Phys 332 HW 0 Solutions

① wangsness 20-1 the magnetization is the dipole moment per unit volume so $M = \text{dipole moment}/\text{atom or molecule} \times \frac{\#\ \text{atoms/molecules}}{m^3}$

$= \mu_e \frac{N}{V}$ $PV = NKT$ $\frac{N}{V} = \frac{P}{KT} = \frac{1.01\times10^{5}\ N/m^2}{1.38\times10^{-23}\ J/K\ (100+273)K}$

$\Rightarrow \underline{M_{Max} \simeq 182\ A/m}$ $= 1.96\times10^{25}$ atoms/molecules $/m^3$



at that distance the dipole approximation will be fine.

at our location, B will be "radial". $B_r = \frac{\mu_0 m}{4\pi}\frac{2\cos\theta}{r^3}$ $\theta = \phi$

$m = M(\text{Volume})$ if all are aligned $B \simeq 10^{-7}(182)(.05)^3 \frac{2}{1^3}$

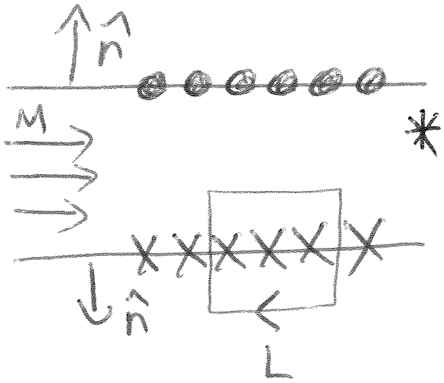
$\underline{\simeq 4.55\times10^{-9}\ T}$

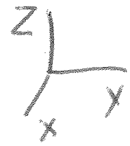
iron $\frac{7870\ Kg}{m^3} \times \frac{6.02\times10^{23}\ \text{atoms}}{\text{mole}} \times \frac{1\ \text{mole}}{.0558\ Kg}$

$\rightarrow N/V = 8.49\times10^{28}\ \text{atoms}/m^3$ $\underline{M_{max} \simeq 7.87\times10^{5}\ A/m}$

so this yields $\underline{B \simeq 1.97\times10^{-5}\ T}$ so clearly the iron can create a field larger than the Earth's field if you are close enough to it

(2) $\vec{J}_M = \vec{\nabla} \times \vec{M}$ so $\underline{\vec{J}_M = \phi}$ $\quad \vec{K}_M = \vec{M} \times \hat{n}$ $\quad \hat{n} = \pm \hat{z}$ $\quad \vec{M} = M_0 \hat{y}$





$\vec{K}_M = M_0 \hat{y} \times (\pm \hat{z})$

$= \pm M_0 \hat{x}$

these K's can now be used to determine B

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{en}$ $\quad 2BL = \mu_0 M_0 L$ $\quad B_{\text{one surface}} = \dfrac{\mu_0 M_0}{2}$

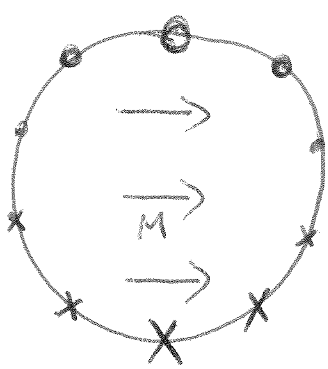
B_{outside} — 2 B's effectively cancel so $\underline{B = \phi}$

*

dipoles arranged so • on top and x on botton $\vec{M}$

B_{inside} - the fields from each $\vec{K}$ add so $\underline{\vec{B}_{\text{inside}} = \mu_0 M_0 \hat{y}}$

(3)



$\vec{M} = M_0 \hat{x}$ $\quad \vec{J} = \vec{\nabla} \times \vec{M} = \underline{\phi}$ $\quad \vec{K} = \vec{M} \times \hat{n}$

$\hat{n} = \hat{\rho}$ so $\vec{K} = M_0 \hat{x} \times (\cos\phi \, \hat{x} + \sin\phi \, \hat{y})$

$\underline{\vec{K} = M_0 \sin\phi \, \hat{z}}$

size of • or X implies magnitude in diagram

if you take the circular cross section shown there is clearly just as much current out as there is current in so the net charge transfered must be zero

③ continued

$$\vec{B} = \frac{\mu_0}{4\pi}\iint \frac{\vec{K}\times\hat{R}}{R^2}\,da \qquad \vec{r} = \phi^{*} \quad \vec{r}' = a\hat{\rho}' + z'\hat{z}$$

$$R = \sqrt{a^2+z^2}$$

$$= \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty}\int_{\phi}^{2\pi} \frac{\hat{z}M_0\sin\phi' \times (a\hat{\rho}' + z'\hat{z})\,d\phi' a\,dz'}{(a^2+z'^2)^{3/2}} \qquad \hat{z}\times\hat{\rho}' = \hat{\phi}'$$

$$\hat{\phi}' = +\cos\phi'\,\hat{y} - \sin\phi'\,\hat{x}$$

$$= \frac{-\mu_0 M_0 a^2}{4\pi}\iint \frac{\hat{\phi}'\sin\phi'\,d\phi' dz'}{(a^2+z'^2)^{3/2}}$$

the current distribution tells you that B must be in $+\hat{x}$.

$\hat{y}$ integral here vanishes $\oint \sin\phi'\cos\phi' d\phi' = \phi$

$$= \frac{\mu_0 M_0 a^2}{4\pi}\int_{-\infty}^{\infty}\int_{\phi}^{2\pi} \frac{\sin^2\phi'\,d\phi' dz'\,\hat{x}}{(a^2+z'^2)^{3/2}}$$

$\langle \sin^2\alpha\rangle = \frac{1}{2}$ so $d\phi'$ integral is $\frac{1}{2}\times 2\pi = \pi$

$$= \frac{\mu_0 M_0 a^2}{4}\int_{-\infty}^{\infty} \frac{dz'\,\hat{x}}{(a^2+z'^2)^{3/2}}$$

$z' = a\tan\theta \quad dz' = a\sec^2\theta\,d\theta$

$$= \frac{\mu_0 M_0 a^2}{4}\int_{-\pi/2}^{\pi/2} \frac{a\sec^2\theta\,d\theta\,\hat{x}}{a^3\sec^3\theta} = \frac{\mu_0 M_0}{4}\sin\theta\Big|_{-\pi/2}^{\pi/2}\hat{x} = \frac{\mu_0 M_0}{4}(1-(-1))\hat{x}$$

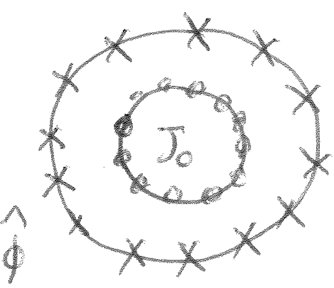
$$\boxed{\vec{B}_{z\,axis} = \frac{\mu_0 M_0}{2}\hat{x}}$$

* since the cylinder is infinite the field must be the same at all z so I might as well put myself at $z = \phi$.

using $\vec{K}$ to find the field off-axis would be quite messy — see problem #5

compare to Wangsness 10-12 which you did earlier. Do you understand the ± difference?

④ $\underline{\rho<a}$ $\oint \vec{H}\cdot d\vec{s} = I_{en}$ $H(2\pi\rho) = J_0(\pi\rho^2)$



so $\underline{\vec{H}_{\rho<a} = \frac{J_0 \rho}{2}\hat{\phi}}$ $\vec{B} = \mu_0 \vec{H}$ so $\underline{\vec{B}_{\rho<a} = \frac{\mu_0 J_0 \rho}{2}\hat{\phi}}$

the wire is not magnetized so $\underline{M = \emptyset}$ (also consistent with $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$)

$\underline{a<\rho<b}$ $\oint \vec{H}\cdot d\vec{s} = I_{en}$ $H(2\pi\rho) = J_0(\pi a^2)$ $\underline{\vec{H} = \frac{J_0 a^2}{2\rho}\hat{\phi}}$

$\vec{B} = \mu \vec{H}$ $\underline{\vec{B} = \frac{\mu J_0 a^2}{2\rho}\hat{\phi}}$ $\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H} = \underline{\frac{J_0 a^2}{2\rho}\hat{\phi}\left(\frac{\mu}{\mu_0} - 1\right)}$

$\underline{\rho>b}$ H is the same as $\underline{\vec{H} = \frac{J_0 a^2}{2\rho}\hat{\phi}}$ $\vec{B} = \mu_0 \vec{H}$ so

and $\underline{\vec{M} = \emptyset}$ (vacuum) $\underline{\vec{B} = \frac{\mu_0 J_0 a^2}{2\rho}\hat{\phi}}$

b) $\vec{J} = \vec{\nabla}\times\vec{M} = \hat{z}\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\,\frac{J_0 a^2}{2\rho}\,\frac{\mu-\mu_0}{\mu_0}\right) = \underline{\emptyset}$

it doesn't matter if they curl around the z axis. What matters is whether they curl locally. Let's zoom in on a region appropriate to this geometry.

If $\vec{\nabla}\times\vec{M} = \emptyset$ then $\oint \vec{M}\cdot d\vec{s} = \emptyset$

on sides 1, 3 $\vec{M}\cdot d\vec{s} = \emptyset$

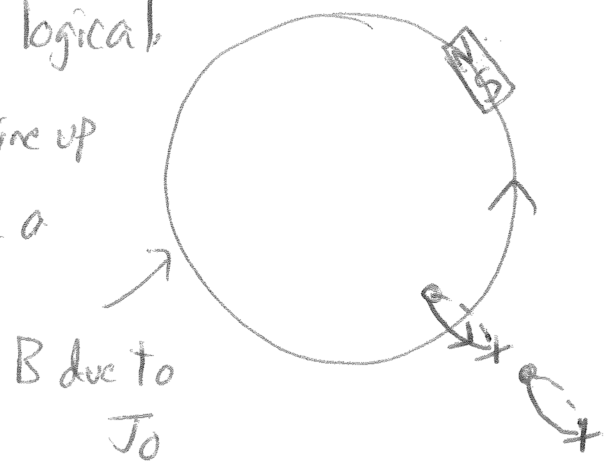
on side 2 $\vec{M}\cdot d\vec{s} < \emptyset$, 4 $\vec{M}\cdot d\vec{s} > \emptyset$

$|\vec{M}| \propto \frac{1}{\rho}$ $|d\vec{s}| \propto \rho$ so the circulation is zero! (analogous to $\vec{\nabla}\cdot\frac{\alpha}{r^2}\hat{r} = \emptyset$ except at origin)

④ continued $\overset{\circ}{K} = \vec{M} \times \hat{n}$ $\hat{n} = -\hat{\rho}$ at $\rho = a$ $\hat{n} = +\hat{\rho}$ at $\rho = b$

$$\underline{\overset{\circ}{K}_{\rho=a} = \frac{J_o a^{\cancel{2}}}{2\cancel{a}}\left(\frac{\mu}{\mu_o}-1\right)\hat{z}} \quad \text{and} \quad \underline{\overset{\circ}{K}_{\rho=b} = \frac{J_o a^2}{2b}\left(\frac{\mu}{\mu_o}-1\right)(-\hat{z})}$$

these are completely logical. the dipoles will try to line up with B so if you imagine a current loop it will have



I out at small ρ and I in at large ρ. The inner edge at $\rho=a$ will look like an outward $\overset{\circ}{K}$ and the outer surface at $\rho=b$ will look like an inward $\overset{\circ}{K}$.

$$\frac{dq}{dt} = \int_S \vec{J}_M \cdot d\vec{a} + \oint_{\rho=a} \overset{\circ}{K}_{\rho=a} \cdot \hat{z}\, a\, d\phi + \oint_{\rho=b} \overset{\circ}{K}_{\rho=b} \cdot \hat{z}\, b\, d\phi$$

$$= \phi + J_o \pi a^2\left(\frac{\mu}{\mu_o}-1\right) - J_o \pi a^2\left(\frac{\mu}{\mu_o}-1\right) = \phi$$ no net charge transfered

this surface current at $\rho = a$ is what makes B larger than it would have been w/o the magnetic material

$(B = \mu I / 2\pi\rho$ vs. $B = \mu_o I / 2\pi\rho)$

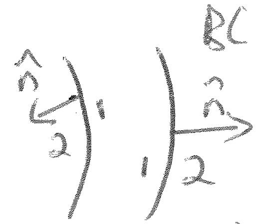
c) <u>normal components</u> $B_{2n} - B_{1n} = \phi$

$H_{2n} - H_{1n} = -(M_{2n} - M_{1n})$

all fields in $\hat{\phi}$ so <u>all</u> normal components are zero $\Longrightarrow$ yes BCs ok

<u>tangential components</u> $\hat{n} \times (\vec{B}_2 - \vec{B}_1) = \mu_0 \vec{K}$

$\hat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{K}_f$



at $\rho = a$) $\hat{n} = -\hat{\rho}$ $\vec{H}_{2\,\rho=a} = \frac{J_0 a}{2}\hat{\phi}$, $\vec{H}_{1\,\rho=a} = \frac{J_0 a}{2}\hat{\phi}$ $K_f = \emptyset$

clearly satisfied ✓

$$-\hat{\rho} \times \left(\frac{\mu_0 J_0 a}{2}\hat{\phi} - \frac{\mu J_0 a}{2}\hat{\phi} \right) = \mu_0 \frac{J_0 a}{2}\left(\frac{\mu}{\mu_0} - 1\right)\hat{z} \quad K_f \mu_0$$

$$-\frac{J_0 a}{2}(\mu_0 - \mu)\,\hat{\rho} \times \hat{\phi} = -\frac{J_0 a}{2}(\mu_0 - \mu)\,\hat{z}$$

again it is satisfied ✓

at $\rho = b$) $\hat{n} = +\hat{\rho}$ $\hat{\rho} \times \left(\frac{J_0 a^2}{2b}\hat{\phi} - \frac{J_0 a^2}{2b}\hat{\phi} \right) = \phi$ ✓

$$\hat{\rho} \times \left(\frac{\mu_0 J_0 a^2}{2b}\hat{\phi} - \frac{\mu J_0 a^2}{2b}\hat{\phi} \right) = \mu_0 \frac{J_0 a^2}{2b}\left(\frac{\mu}{\mu_0} - 1\right)(-\hat{z}) \quad K_f$$

$$\hat{\rho} \times \hat{\phi}\, \frac{J_0 a^2}{2b}(\mu_0 - \mu)$$

$$\parallel$$

$$\frac{J_0 a^2}{2b}(\mu_0 - \mu)\,\hat{z} = \frac{J_0 a^2}{2b}(\mu - \mu_0)(-\hat{z})$$ ✓

so all BCs satisfied

4-continued - part d $\quad U_M = \frac{1}{2}\vec{H}\cdot\vec{B} = \frac{1}{2}\mu H^2$

$$U_M = \iiint u_m \, d\tau = \int_{\emptyset}^{a}\int_{\emptyset}^{2\pi}\int_{\emptyset}^{L} \frac{1}{2}\mu_0 \frac{J_0^2 \rho^2}{4} \rho \, d\rho \, d\phi \, dz + \int_{a}^{b}\int_{\emptyset}^{2\pi}\int_{\emptyset}^{L} \frac{1}{2}\mu \frac{J_0^2 a^4}{4\rho^2} \rho \, d\rho \, d\phi \, dz$$

$$+ \int_{b}^{\rho}\int_{\emptyset}^{2\pi}\int_{\emptyset}^{L} \frac{1}{2}\mu_0 \frac{J_0^2 a^4}{4\rho^2} \rho \, d\rho \, d\phi \, dz$$

$$= \frac{1}{2} J_0^2 \frac{1}{4} 2\pi L \left[\mu_0 \frac{1}{4} a^4 + \mu a^4 \ln\frac{b}{a} + \mu_0 a^4 \ln\frac{\rho}{b}\right]$$

or $$U_{\text{in cylindrical region}} = \frac{J_0^2 \pi L a^4}{4}\left[\frac{\mu_0}{4} + \mu \ln\frac{b}{a} + \mu_0 \ln\frac{\rho}{b}\right]$$

5) $\vec{M} = M_0 \hat{x} \quad \rho_M = -\vec{\nabla}\cdot\vec{M} = \underline{\emptyset} \quad \sigma_M = \vec{M}\cdot\hat{n} = M_0 \hat{x}\cdot\hat{\rho}$

$$\underline{\sigma_M = M_0 \cos\phi}$$

since there are no free currents

$\vec{\nabla}\times\vec{H} = \emptyset$ so $\vec{H} = -\vec{\nabla}\Phi_M$

now $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ so $\vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M}$ so $\vec{\nabla}\cdot\vec{H} = \emptyset$

so $\vec{\nabla}\cdot(-\vec{\nabla}\Phi_M) = \emptyset$ or $\nabla^2\Phi_M = \emptyset$

now $\Phi_M = \frac{1}{4\pi}\int_{S'} \frac{\sigma_M \, da'}{R}$ but that is messy

let's just solve Laplace's equation

since the cylinder is ∞ ϕ_m cannot be a function of z

so the general solution in that case is :

$$\phi_m = A + B\ln\rho + \sum_{m=1}^{\infty}\left(A_m\rho^m + \frac{B_m}{\rho^m}\right)\left(C_m\cos m\phi + D_m\sin m\phi\right)$$

since $\sigma_m = M_0\cos\phi$ ϕ_m can only have terms with $m=1$

this will be a dipole far away as we will see

so $\phi_m = \left(A\rho + \frac{B}{\rho}\right)\cos\phi$ now use what we know about ϕ

$\phi_{\rho\to\infty}$ must $=\emptyset$ so ϕ_m (outside) $= \frac{B}{\rho}\cos\phi$

$\phi_{\rho\to\emptyset}$ must be finite so ϕ_m(inside) $= A\rho\cos\phi = Ax$

ϕ is continuous at the boundary so $\frac{B}{a}\cos\phi = Aa\cos\phi$

so $B = Aa^2$

so $\phi_m(\rho>a) = \frac{Aa^2}{\rho}\cos\phi$

$\phi_m(\rho<a) = A\rho\cos\phi$

now $\vec{H} = -\vec{\nabla}\phi$

$\vec{\nabla} = \hat{\rho}\frac{\partial}{\partial\rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial}{\partial\phi}$

$\vec{H}_{\rho<a} = -A\cos\phi\,\hat{\rho} + A\sin\phi\,\hat{\phi}$

$\vec{H}_{\rho>a} = \frac{Aa^2}{\rho^2}\cos\phi\,\hat{\rho} + \frac{Aa^2}{\rho^2}\sin\phi\,\hat{\phi}$

now use BCs on H to find A

5-continued

$\hat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{K}_f$ $\quad \hat{n} = \hat{\rho}$ $\quad \vec{K}_f = \emptyset$

the $\hat{\rho}$ parts of H don't matter since $\hat{\rho} \times \hat{\rho} = \emptyset$

so $\hat{\rho} \times \left(\frac{Aa^2}{a^2} \sin\phi \, \hat{\phi} - A \sin\phi \, \hat{\phi} \right) = \emptyset$

this ~~a~~ is already satisfied. This would have told us that $B = Aa^2$ if we hadn't used $\Phi_{in} = \Phi_{out}$ at $\rho = a$.

$H_{2n} - H_{1n} = -(M_{2n} - M_{1n})$ $\quad \frac{Aa^2}{a^2} \cos\phi - (-A\cos\phi) = -(\emptyset - M_0 \cos\phi)$

$2A\cos\phi = M_0 \cos\phi \longrightarrow A = M_0/2$

so $\vec{H}_{\rho<a} = -\frac{M_0}{2} (\cos\phi \, \hat{\rho} - \sin\phi \, \hat{\phi})$ $\quad$ with $\hat{\rho} = \cos\phi \, \hat{x} + \sin\phi \, \hat{y}$ and $\hat{\phi} = \cos\phi \, \hat{y} - \sin\phi \, \hat{x}$

$= \hat{x}$

so $\boxed{\vec{H}_{\rho<a} = -\frac{M_0}{2} \hat{x}}$ $\quad$ now $\vec{H}_{loc} = -N_m \vec{M}$ so demagnetizing factor $= \frac{1}{2}$

$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ $\quad -\frac{M_0}{2} \hat{x} = \frac{\vec{B}}{\mu_0} - M_0 \hat{x}$ $\quad$ so $\boxed{\vec{B}_{\rho<a} = \frac{\mu_0 M_0}{2} \hat{x}}$ agrees w/ previous result on z axis

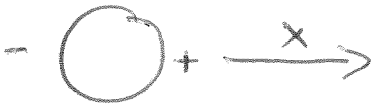
outside $\vec{B} = \mu_0 \vec{H}$

$\vec{H}_{\rho>a} = \frac{M_0 a^2}{2\rho^2} (\cos\phi \, \hat{\rho} + \sin\phi \, \hat{\phi})$

so $\vec{B}_{\rho>a} = \frac{\mu_0 M_0 a^2}{2\rho^2} (\cos\phi \, \hat{\rho} + \sin\phi \, \hat{\phi})$

this is a dipole field

"normal point dipole": $\vec{B} = \left(\frac{\mu_0 m}{4\pi}\right)\left(\frac{2\cos\theta\,\hat{r}}{r^3} + \frac{\sin\theta\,\hat{\theta}}{r^3}\right)$
on z axis

we have  so our dipole is along X
since $\cos(-\phi) = \cos\phi$, $\cos\phi$ plays the same role as $\cos\theta$ above, Factor of 2 is different.

since $\sin\theta > 0$ for $z > 0$ and $\hat{\theta}$ points along lines of longitude <u>and</u> since $\sin\phi$ flips sign at $\phi = 0$ while $\hat{\phi}$ is ccw the $\sin\theta\,\hat{\theta}$ and $\sin\phi\,\hat{\phi}$ terms do exactly the same thing! Point dipole has r^3 while this is ρ^2. That is equivalent to saying that $E_{\text{point charge}} \propto 1/r^2$ while $E_{\infty\text{ line}} \propto 1/\rho$. Both are monopoles but distance dependence is different. point dipole has $1/r^3$

while ∞ line dipole has $1/\rho^2$.

6 this is analogous to the dielectric sphere in an initially uniform $\vec{E}_0$ as in the previous problem $\vec{\nabla}\times\vec{H} = 0$ and $\vec{\nabla}\cdot\vec{H} = 0$

here also $\vec{\nabla}\cdot\vec{B} = 0 = \vec{\nabla}\cdot(\mu\vec{H}) = \mu\vec{\nabla}\cdot\vec{H}$ so

reminder: $B = \mu H = K_m \mu_0 H$ thus $\nabla^2 \Phi_m = 0$

so $K_m = \mu/\mu_0$ and $\vec{H} = -\vec{\nabla}\Phi_m$

6-continued

since this is axisymmetric we can start with

$$\phi_m(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

which is the general solution to $\nabla^2\phi = 0$ in that geometry

BCs

1) at points far outside the sphere $\vec{H} = H_0\hat{z}$ $\quad \vec{H} = -\vec{\nabla}\phi_m$

so $\phi_m = -H_0 z = -H_0 \underline{r\cos\theta}$

the $\cos\theta$ tells us that we can only have P_1 (by orthogonality)

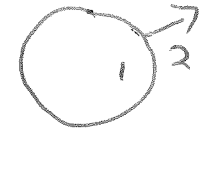
and the r also tells us that $l=1$

so $\phi_{r>a} = \left(A_1 r + \dfrac{B_1}{r^2}\right)\cos\theta = -H_0 r\cos\theta$ for larger r

B_1 still undetermined $\qquad$ so $\underline{A_1 = -H_0}$

2) at $r=0$, ϕ must not diverge since there are no magnetic poles there

$\phi_{r<a} = \left(A_1' r + \dfrac{B_1'}{r^2}\right)\cos\theta$ $\quad$ so $B_1' = 0$, A_1' still undetermined



3) $\hat{n}\cdot(\vec{B}_2 - \vec{B}_1) = 0$ so $B_{2n} = B_{1n}$ $\quad B = \mu H$

so $\mu_0(-\nabla\phi_{r>a})\Big|_{r=a} = \mu(-\nabla\phi_{r<a})\Big|_{r=a}$ $\quad \vec{\nabla}\phi_{r>a}\Big|_a = K_m \vec{\nabla}\phi_{r<a}\Big|_a$

$\vec{\nabla} = \hat{r}\dfrac{\partial}{\partial r} + \hat{\theta}\dfrac{1}{r}\dfrac{\partial}{\partial\theta} + \hat{\phi}\dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial\phi}$ $\quad -H_0\cancel{\cos\theta} - \dfrac{2B_1}{a^3}\cancel{\cos\theta} = K_m A_1'\cancel{\cos\theta}$

I only want $\hat{r}$ part here

4) $\hat{n} \times (\vec{H}_2 - \vec{H}_1) = \phi$ (no free current) $\hat{n} = \hat{\rho}$

or $H_{2\theta} = H_{1\theta}$ so this time I keep the $\hat{\theta}$ term from

the gradient $\nabla\phi_{r<a}\Big|_{\hat{\theta}\text{ part},a} = \nabla\phi_{r>a}\Big|_{\hat{\theta}\text{ part},a}$

$$-A_1' \cancel{\sin\theta} = \cancel{A_1} + H_0 \cancel{\sin\theta} - \frac{B_1}{a^3}\cancel{\sin\theta}$$

5) $\phi_{r<a}\Big|_a = \phi_{r>a}\Big|_a$ this reproduces the result of 4)

algebra

$$H_0 + \frac{2B_1}{a^3} = -K_m A_1'$$ subtract them to get

$$H_0 - \frac{B_1}{a^3} = -A_1' \qquad \frac{3B_1}{a^3} = (-K_m + 1)A_1'$$

or $B_1 = \dfrac{-(K_m - 1)A_1' a^3}{3}$ $\qquad H_0 + \dfrac{1}{a^3}\dfrac{(K_m-1)A_1' a^3}{(3)} = -A_1'$

$$H_0 = -A_1'\left(1 + \frac{K_m - 1}{3}\right) = -A_1'\left(\frac{2+K_m}{3}\right) \qquad \underline{A_1' = -\frac{3}{2+K_m} H_0}$$

so $B_1 = \dfrac{+(K_m-1)a^3}{3}\dfrac{3}{2+K_m} H_0 = \underline{\dfrac{K_m - 1}{2+K_m} a^3 H_0 = B_1}$

so $\phi_{r<a} = -\dfrac{3}{2+K_m} H_0 r\cos\theta$ and $\phi_{r>a} = -H_0 r\cos\theta + \dfrac{K_m - 1}{2+K_m}\dfrac{a^3}{r^2} H_0\cos\theta$

6-continued

so $\vec{H} = -\left(\hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\right)\Phi_M$

so for $r<a$ $\quad H_r = \frac{3}{2+K_m} H_0 \cos\theta$ and $H_\theta = \frac{-3}{2+K_m} H_0 \sin\theta$

now $\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta} = \hat{z}$ so $\boxed{\vec{H}_{r<a} = \frac{3}{2+K_m}\vec{H}_0}$

for $r>a$

$$H_r = H_0\cos\theta\left(1 + 2\frac{K_m-1}{2+K_m}\frac{a^3}{r^3}\right)$$

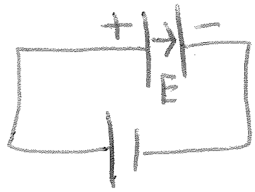
and

$$H_\theta = -H_0\sin\theta\left(1 - \frac{K_m-1}{2+K_m}\frac{a^3}{r^3}\right)$$

so $\vec{H}_{r>a} = \vec{H}_0$ + dipole field

see 19-24.
w/ $\frac{\mu_0 M}{4\pi} = \frac{K_m-1}{2+K_m} H_0 a^3$

(7) wangsness 21-1



viewed from the right

$\vec{\nabla}\times\vec{H} = \vec{J}_f + \dot{\vec{D}}$ $\quad J_f = \phi$ between the plates

so $\oint \vec{H}\cdot d\vec{s} = \iint \frac{\partial \vec{D}}{\partial t}\cdot d\vec{a}$ $\quad$ since the oscillation is slow, the system has time to respond so $E = \frac{\mathcal{E}}{d}$

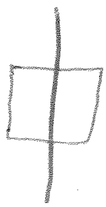
with the circular symmetry in this system, the amperian loop must be a circle in the plane between the plates

$D = \epsilon_0 E = \frac{\mathcal{E}\epsilon_0}{d_0 + d_1\sin\omega t}$ $\quad \dot{D} = \frac{-\mathcal{E}\epsilon_0 d_1 \omega\cos\omega t}{(d_0 + d_1\sin\omega t)^2}$ $\quad$ *voltage always $\mathcal{E}$

$$H(2\pi\rho) = \frac{-\epsilon\,\epsilon_0 d_1 \omega \cos\omega t}{(\qquad)^2}\pi\rho^2 \quad \text{or} \quad \boxed{\vec{H} = \frac{\epsilon_0\,\epsilon\, d_1 \omega \cos\omega t\,\rho}{2(d_0 + d_1 \sin\omega t)^2}(-\hat{\phi})}$$

b) now the voltage is changing as is "d" plates effectively infinite

$\vec{\nabla}\cdot\vec{D} = \rho_f$



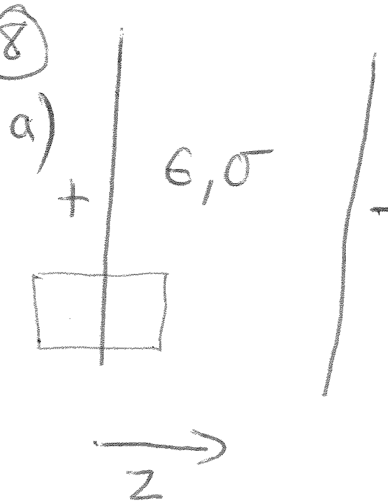
$\iint \vec{D}\cdot d\vec{a} = Q_{en}$ $\quad 2DA = \sigma_f A \quad D = \frac{\sigma_f}{2}$ but there are two plates so $D = \sigma_f \quad D = \epsilon_0 E$ so $E = \frac{\sigma_f}{\epsilon_0}$

$\vec{\nabla}\times\vec{H} = \cancel{\vec{J}_f} + \dot{\vec{D}} \quad \dot{\vec{D}} = \emptyset$ so $\vec{\nabla}\times\vec{H} = \emptyset \implies \underline{H = \emptyset}$

(keep in mind that $\vec{\nabla}\times\vec{H} = \emptyset$ doesn't, in general, mean that $H = \emptyset$. It does work in this case $H(2\pi\rho) = \emptyset$ (cylindrical symmetry here) so $H = \emptyset$)

8)

a)



$I = \frac{dq}{dt} = \frac{d}{dt}(\sigma_{cn} A) = \dot{\sigma}_{cn} A$ so $J = \dot{\sigma}_{cn}$

$\oiint \vec{D}\cdot d\vec{a} = Q_{en} \quad 2DA' = \sigma_{cn} A' \quad D = \frac{\sigma_{cn}}{2}$

there are two plates so $D = \sigma_{cn} \quad D = \epsilon E$

so $E = \frac{\sigma_{cn}}{\epsilon} \quad J = \sigma E$ so $J = \frac{\sigma\,\sigma_{cn}}{\epsilon}$

both must represent currents to the right. Now $\dot{\sigma}_{cn} < \emptyset$

so I must write this as $-\dot{\sigma}_{cn} = \frac{\sigma\,\sigma_{cn}}{\epsilon}$

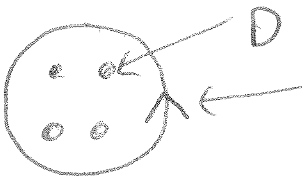
or $\displaystyle\int_{\sigma_{cn\emptyset}}^{\sigma_{cn}} \frac{d\sigma_{cn}}{\sigma_{cn}} = -\frac{\sigma}{\epsilon}\int_{\emptyset}^{t} dt \quad \ln\frac{\sigma_{cn}}{\sigma_{cn\emptyset}} = -\frac{\sigma}{\epsilon}t \implies \boxed{\sigma_{cn}(t) = \sigma_{cn\emptyset}\, e^{-\sigma t/\epsilon}}$

8-continued: σ is in the numerator of the exponent since large conductivities lead to large currents so σ_{ch} decays quickly. Similarly, a large ϵ leads to a small electric field so the driving force for the current is small so σ_{ch} decays slowly. Thus, ϵ must be in the denominator. $\vec{\nabla} \times \vec{H} = \vec{J}_f + \dot{\vec{D}}$

b) the real current in $\hat{z}$ makes $\vec{H}$ in $\hat{\phi}$

since $\dot{\sigma}_{ch} < 0$ $\dot{D}$ is in $-\hat{z}$ so that makes $\dot{H}$ in $-\hat{\phi}$

(there is a decreasing flux of $\dot{D}$ so ampere's law says that H must curl clockwise) $\oint \vec{H} \cdot d\vec{s} = \iint \vec{J}_f \cdot da + \iint \dot{\vec{D}} \cdot da$



direction for

$$J_f = \frac{\sigma \sigma_{ch}}{\epsilon} \qquad \dot{D} = \dot{\sigma}_{ch}$$

$$H(2\pi\rho) = \frac{\sigma \sigma_{ch}}{\epsilon}\pi\rho^2 + \dot{\sigma}_{ch}\pi\rho^2$$

remember that $\dot{\sigma}_{ch} < 0$ so these are in opposite directions

but we already showed that

$$\dot{\sigma}_{ch} = -\frac{\sigma \sigma_{ch}}{\epsilon}$$ so the right side is zero!

So the "displacement current" effectively cancels the real current and $H = 0$