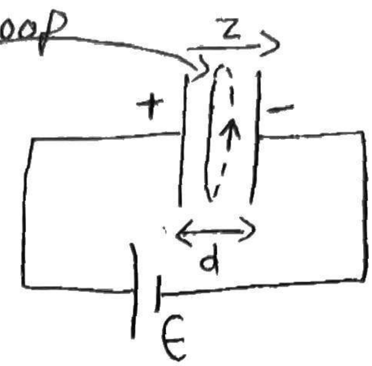
HW #1 Solutions

Wangsness 21-1



a) $\vec{\nabla} \times \vec{H} = \vec{J}_f + \dot{\vec{D}}$ $\quad \vec{J}_f = \phi$ between plates

$$\oint \vec{H} \cdot d\vec{s} = \iint \frac{\partial \vec{D}}{\partial t} \cdot d\vec{a}$$

since the plates are circular, symmetry demands that the loop be a circle in the plane of the plates

since the oscillation is slow the system has time to respond so $E = \mathcal{E}/d$

$$D = \epsilon_0 E = \frac{\epsilon_0 \mathcal{E}}{d_0 + d_1 \sin\omega t}$$

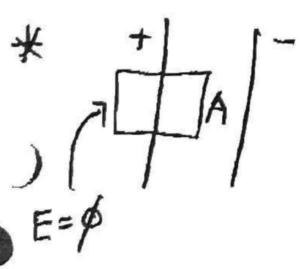
so $\dot{D} = (-1) \dfrac{\epsilon_0 \mathcal{E}}{(\quad)^2} (+d_1 \omega \cos\omega t)$ — which is constant over the surface

$$H(2\pi\rho) = \frac{-\epsilon_0 \mathcal{E} d_1 \omega \cos\omega t}{(d_0 + d_1 \sin\omega t)^2} \pi\rho^2$$

or $\boxed{\vec{H} = \dfrac{\epsilon_0 \mathcal{E} d_1 \omega \cos\omega t \, \rho}{2(d_0 + d_1 \sin\omega t)^2} (-\hat{\phi})}$

b) since the plates are effectively infinitely large, the electric field between them is not a function of distance* – so

$\dot{E} = \dot{D} = \phi \quad \rightarrow \quad \underline{H = \phi}$



$\vec{\nabla} \cdot \vec{D} = \rho_f \quad \vec{\nabla} \cdot \epsilon_0 \vec{E} = \rho_f \quad \oiint \vec{E} \cdot d\vec{a} = Q_{en}/\epsilon_0$

$EA = \dfrac{\sigma A}{\epsilon_0} \qquad E = \sigma/\epsilon_0$

Wangsness 21-7 $\quad \epsilon = \epsilon(x,y,z), \quad \mu = \mu(x,y,z) \qquad D = \epsilon E, \quad B = \mu H$

) $\vec{\nabla}\cdot\vec{D} = \rho_f \qquad \vec{\nabla}\cdot(\epsilon\vec{E}) = \rho_f \qquad \vec{\nabla}\epsilon\cdot\vec{E} + \epsilon\,\vec{\nabla}\cdot\vec{E} = \rho_f$

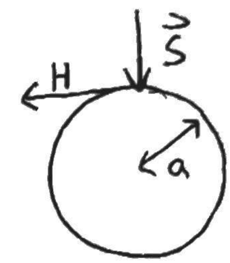
$\vec{\nabla}\cdot\vec{B} = \emptyset$ and $\vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t}$ unchanged

or $\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho_f}{\epsilon} - \frac{1}{\epsilon}\vec{\nabla}\epsilon\cdot\vec{E}}$

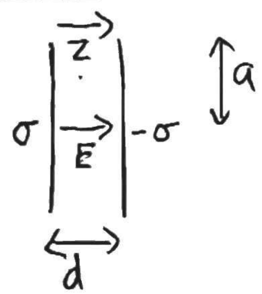
$\vec{\nabla}\times\vec{H} = \vec{J}_f + \frac{\partial\vec{D}}{\partial t} \qquad \vec{\nabla}\times\frac{\vec{B}}{\mu} = \vec{J}_f + \frac{\partial}{\partial t}(\epsilon\vec{E})$

$\vec{\nabla}\frac{1}{\mu}\times\vec{B} + \frac{1}{\mu}\vec{\nabla}\times\vec{B} = \vec{J}_f + \epsilon\frac{\partial\vec{E}}{\partial t} \qquad -\frac{1}{\mu^2}\vec{\nabla}\mu\times\vec{B} + \frac{1}{\mu}\vec{\nabla}\times\vec{B} = \vec{J}_f + \epsilon\frac{\partial\vec{E}}{\partial t}$

or $\boxed{\vec{\nabla}\times\vec{B} = \mu\left[\vec{J}_f + \epsilon\frac{\partial\vec{E}}{\partial t} + \frac{1}{\mu^2}\vec{\nabla}\mu\times\vec{B}\right]}$



) Wangsness 21-9



see text for details:

$\Rightarrow \vec{E} = \frac{\sigma}{\epsilon_0}\hat{z}, \quad \vec{H} = \frac{I}{2\pi a}\hat{\phi}$

$\vec{S} = \vec{E}\times\vec{H} = \frac{\sigma I}{2\pi a\epsilon_0}(-\hat{\rho})$

$\rightarrow$ energy appears to flow inwards

$P = \iint S\,da = \int_0^{2\pi}\int_0^{d}\frac{\sigma I}{2\pi a\epsilon_0}\,a\,d\phi\,dz$

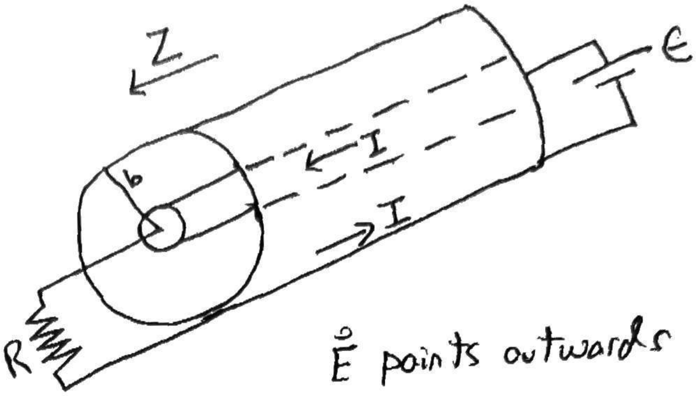
$\Rightarrow \boxed{P = \sigma I d/\epsilon_0}$

$U = \frac{Q^2}{2C} = \frac{(\sigma A)^2 d}{2\epsilon_0 A} = \frac{\sigma^2 A d}{2\epsilon_0}$ so $\dot{U} = \frac{2\sigma\dot{\sigma}Ad}{2\epsilon_0}$

now $\dot{\sigma}A = \dot{Q} = I$ so $\boxed{\dot{U} = \sigma I d/\epsilon_0 = P}$ ✓

Wangsness 21-10

) $\vec{S} = \vec{E} \times \vec{H}$



there is a ΔV between $\rho = a$ and $\rho = b$ so you could get $\vec{E}$ using Laplace's eq.

$\vec{E}$ points outwards

Since there is cylindrical symmetry and an ∞ line, $E \propto \frac{1}{\rho}$ (just like E from an ∞ line of charge)

so $\vec{E} = \frac{K}{\rho}\hat{\rho}$ $\quad |\Delta V| = \int_a^b \vec{E} \cdot d\rho\,\hat{\rho} = \int_a^b \frac{K}{\rho} d\rho = K \ln b/a = \epsilon$

so $K = \epsilon / \ln b/a$

$\vec{\nabla} \times \vec{H} = \vec{J}_f + \cancel{\frac{\partial \vec{D}}{\partial t}}$ $\quad \oint \vec{H} \cdot d\vec{s} = I_{en}$ $\quad H(2\pi\rho) = I$ $\quad \vec{H} = \frac{I}{2\pi\rho}\hat{\phi}$

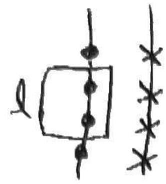
) $\vec{S} = \frac{\epsilon}{\ln b/a} \frac{1}{\rho} \frac{I}{2\pi\rho} \hat{\rho} \times \hat{\phi}$ $\quad \vec{S} = \frac{I\epsilon\,\hat{z}}{2\pi\rho^2 \ln b/a}$ — between $\rho = a$ and $\rho = b$ only

$P = \int_0^{2\pi}\int_a^b \vec{S} \cdot \rho\, d\rho\, d\phi\, \hat{z} = \frac{I\epsilon}{2\pi \ln b/a} 2\pi \ln b/a = \boxed{I\epsilon}$ $\quad$ w/ $I = \epsilon/R$ $= \frac{\epsilon^2}{R}$

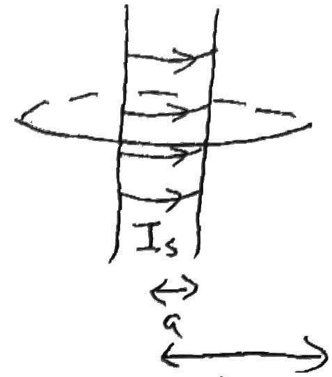
$I\epsilon$ is the same as the energy supplied to the resistor

reverse connection inner conductor is now at lower voltage so $\vec{E}$ in $-\hat{\rho}$. I reverses direction so $\vec{H}$ in $-\hat{\phi}$ so $\vec{S}$ is still in $\hat{z}$. So either way energy appears to flow from the battery towards the resistor

Griffiths 8-9



B = ∅ outside solenoid



$\vec{\nabla}\times\vec{H} = \vec{J}_f + \cancel{\dot{\vec{D}}}$

a) $\oint \vec{H}\cdot d\vec{s} = I_{en}$ $\quad Hl = nlI_s \quad H = nI_s \quad B = \mu_0 n I_s$

$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \oint \vec{E}\cdot d\vec{s} = |\Delta V| = -\frac{d\Phi_B}{dt}$

$\Delta V = -\frac{d}{dt}\left(\mu_0 n I_s \pi a^2\right) = I_r R$ so $\boxed{I_r = -\frac{\mu_0 n \pi a^2}{R}\frac{dI_s}{dt}}$

b) $E(2\pi a) = -\mu_0 n \pi a^2 \frac{dI_s}{dt}$ getting E at edge of solenoid

$\vec{E} = -\frac{1}{2}\mu_0 n a \frac{dI_s}{dt}\hat{\phi}$ remember that $\dot{I} < 0$ so $\vec{E}$ is in $+\hat{\phi}$ which is consistent with Lenz' Law

$\vec{B}$ - from text $= \frac{\mu_0 I_r}{2}\frac{b^2}{(b^2+z^2)^{3/2}}\hat{z} \quad \vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \quad -\hat{\phi}\times\hat{z} = -\hat{\rho}$

$$P = \iint \vec{S}\cdot d\vec{a} = \int_0^{2\pi}\int_{-\infty}^{\infty} -\frac{1}{4}\mu_0 I_r \frac{dI_s}{dt}\frac{ab^2 n}{(b^2+z^2)^{3/2}}\, a\, d\phi\, dz$$

$$= \underbrace{\left[-\frac{1}{2}\pi\mu_0 a^2 b^2 n I_r \dot{I}_s\right]}_{=A}\int_{-\infty}^{\infty}\frac{dz}{(b^2+z^2)^{3/2}}$$

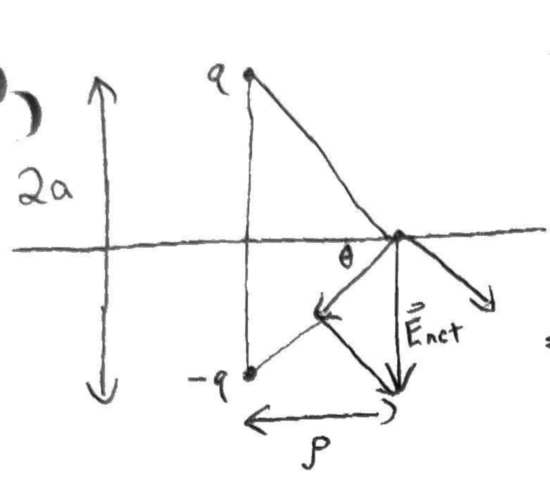
$z = b\tan\theta \quad dz = b\sec^2\theta\, d\theta$

$1 + \tan^2\theta = \sec^2\theta$

$$= A\int_{-\pi/2}^{\pi/2}\frac{b\sec^2\theta\, d\theta}{b^3\sec^3\theta} = \frac{A}{b^2}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \frac{A}{b^2}\sin\theta\Big]_{-\pi/2}^{\pi/2} = \frac{2A}{b^2}$$

$$= \frac{2}{b^2}\underbrace{\left[\frac{-\mu_0 n\pi a^2}{R}\dot{I}_s\right]}_{I_r}\left(-\frac{1}{2}b^2 I_r\right)R = \underline{I_r^2 R} \checkmark$$

8-4b



vertical components add

$$E_z = -2\frac{q\,\hat{z}}{4\pi\epsilon_0(\rho^2+a^2)}\sin\theta$$

$$= -\frac{2q\hat{z}}{4\pi\epsilon_0(\rho^2+a^2)}\frac{a}{\sqrt{\rho^2+a^2}} = \frac{-qa\hat{z}}{2\pi\epsilon_0(\rho^2+a^2)^{3/2}}$$

so $E^2 = E_z^2 = \left(\frac{qa}{2\pi\epsilon_0}\right)^2\frac{1}{(\rho^2+a^2)^3}$

so $T_{zz} = \epsilon_0\left(E_z^2 - \frac{1}{2}E^2\right)$

$$= \frac{1}{2}\frac{q^2a^2}{4\pi^2\epsilon_0}\frac{1}{(\rho^2+a^2)^3}$$

so $\vec{F} = \int_0^{2\pi}\int_0^{\infty} T_{zz}\,da_z$

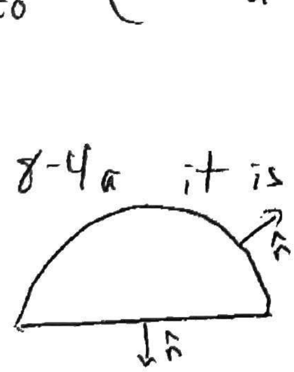
$$= -\int_0^{2\pi}\int_0^{\infty}\frac{1}{2}\frac{q^2a^2}{4\pi^2\epsilon_0}\frac{1}{(\rho^2+a^2)^3}\rho\,d\rho\,d\phi\,\hat{z}$$

$$= \frac{-1\hat{z}}{4\pi\epsilon_0}q^2a^2\int_0^{\infty}\frac{\rho\,d\rho}{(\rho^2+a^2)^3} = \frac{-q^2a^2\hat{z}}{4\pi\epsilon_0}\left(-\frac{1}{4}\right)\frac{1}{(\rho^2+a^2)^2}\Bigg|_0^{\infty}$$

$$= \frac{+q^2a^2\hat{z}}{16\pi\epsilon_0}\left(\emptyset - \frac{1}{a^4}\right) = \frac{-q^2\hat{z}}{16\pi\epsilon_0 a^2} = \frac{-q^2\hat{z}}{4\pi\epsilon_0(2a)^2}$$

as in 8-4a it is really a closed integral

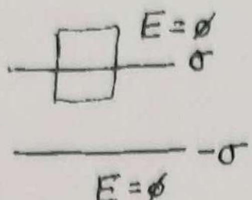
$T_{xy} = T_{yx} = T_{xz} = T_{zx} = T_{yz} = T_{zy} = \emptyset$



the $\int$ over the hemisphere is $\emptyset$ since $T_{ij} = \emptyset$ infinitely far from the charges

also $T_{xx} = T_{yy} \neq \emptyset$ but since da_x and $da_y = \emptyset$, they do not lead to any forces

Griffiths 8-5



$\uparrow z$ so $E_x = E_y = \emptyset$

$\vec{B} = \emptyset$

a) $\vec{\nabla} \cdot \vec{D} = \rho_f \quad \vec{\nabla} \cdot \epsilon_0 \vec{E} = \rho_f \quad \oiint \vec{E} \cdot d\vec{a} = Q_{en}/\epsilon_0 \quad EA = \sigma A/\epsilon_0$

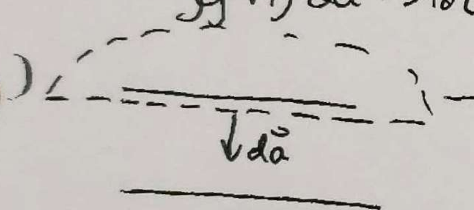
$T_{ij} = \epsilon_0 (E_i E_j - \frac{1}{2}\delta_{ij} E^2) + \underset{(=\emptyset)}{\cdots}$ $\qquad E = \sigma/\epsilon_0$ so $\vec{E} = \frac{-\sigma}{\epsilon_0}\hat{z}$ between plates

$T_{xx} = \epsilon_0(\emptyset - \frac{1}{2}E^2) = T_{yy} \quad T_{zz} = \epsilon_0(E^2 - \frac{1}{2}E^2) = \frac{1}{2}\epsilon_0 E^2$

$T_{xy} = T_{yx} = T_{xz} = T_{zx} = T_{yz} = T_{zy} = \emptyset$

$\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \frac{\sigma^2}{\epsilon_0^2} = \frac{\sigma^2}{2\epsilon_0}$ $\qquad \boxed{T_{ij} = \frac{\sigma^2}{2\epsilon_0}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$

b) $\vec{F} = \oiint T_{ij} \cdot d\vec{a} - \mu_0 \epsilon_0 \frac{d}{dt}\iiint \vec{S}\, d\tau \quad \vec{S} = \emptyset$ so $\vec{F} = \oiint T_{ij} \cdot d\vec{a}$



∞ hemisphere just below plate

$\int$ over curved $= \emptyset$ since $T_{ij} = \emptyset$ there

$\vec{F} = \frac{\sigma^2}{2\epsilon_0}\int_0^{"\infty"}\int_0^{2\pi} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ -\rho\, d\rho\, d\phi\, \hat{z} \end{pmatrix} = \frac{-\sigma^2 \hat{z}}{2\epsilon_0}\iint \rho\, d\rho\, d\phi$ the result will be ∞ but we want F/A

so $\vec{F} = -\frac{\sigma^2}{2\epsilon_0} A\,\hat{z}$ $\qquad \boxed{\vec{f} = \frac{\vec{F}}{A} = \frac{-\sigma^2}{2\epsilon_0}\hat{z}}$

c) T_{ij} is $\vec{p}$ flux in direction i on/~~through~~ a surface whose $\perp$ is in j

so it's just asking for T_{zz} $= \boxed{\sigma^2/2\epsilon_0}$

d) $\frac{T_{zz}\, da_z}{|da_z|}$ gives that but da_z is negative here

$\Rightarrow \boxed{-\frac{\sigma^2}{2\epsilon_0}\hat{z} = \vec{f}}$ as above