HW#3 Solutions

Griffith's 9-1

f_1: $\frac{\partial^2 f_1}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f_1}{\partial t^2}$ $\quad f_1 = Ae^{-b(z-vt)^2}$ $\quad \frac{\partial f_1}{\partial z} = A(-2b)(z-vt)e^{-b(z-vt)^2}$

$$\frac{\partial^2 f_1}{\partial z^2} = -2bA(z-vt)(-2b)(z-vt)e^{-b(z-vt)^2} - 2bAe^{-b(z-vt)^2}$$

$$= -2bAe^{-b(z-vt)^2}\left[1-2b(z-vt)^2\right]$$

$\frac{1}{v^2}\frac{\partial f}{\partial t} = \frac{1}{v^2}(-2bA)(-v)(z-vt)e^{-b(z-vt)^2}$ $\quad \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = +\frac{1}{v}2bA\left[(z-vt)^2(-2b)e^{(\;)}(-v) - ve^{(\;)}\right]$

or $\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 2bAe^{-b(z-vt)^2}\left[2b(z-vt)^2-1\right]$ ✓

f_2: $f_2 = A\sin(bz-bvt)$ $\quad \frac{\partial f_2}{\partial z} = Ab\cos(bz-bvt)$ $\quad \frac{\partial^2 f}{\partial z^2} = -Ab^2\sin(bz-bvt)$

$\frac{1}{v^2}\frac{\partial f_2}{\partial t} = \frac{1}{v^2}A(-bv)\cos(bz-bvt)$ $\quad \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = -\frac{Ab}{v}(-bv)(-1)\sin(bz-bvt)$

$$= -Ab^2\sin(bz-bvt) \checkmark$$

f_3: $f_3 = \frac{A}{b(z-vt)^2+1}$ $\quad \frac{\partial f_3}{\partial z} = \frac{-2bA(z-vt)}{[b(z-vt)^2+1]^2}$

$$\frac{\partial^2 f_3}{\partial z^2} = -2bA\left[\frac{1}{[\quad]^2} - \frac{2(z-vt)(-2b)(z-vt)}{[\quad]^3}\right] = \frac{-2Ab}{[\;]^2} + \frac{8b^2A(z-vt)^2}{[\quad]^3}$$

$$\frac{1}{v^2}\frac{\partial f_3}{\partial t} = \frac{-2b(z-vt)(-v)A}{[\quad]^2 v^2} = \frac{2bA}{v}\left[\frac{-v}{[\;]^2} + \frac{(z-vt)(+2b)(-v)(z-vt)(-2)}{[\quad]^3}\right]$$

$$= \frac{-2bA}{[\;]^2} + \frac{8b^2A(z-vt)^2}{[\quad]^3} \checkmark$$

f_4: $f_4 = A e^{-b(bz^2+vt)}$ $\frac{\partial f_4}{\partial z} = -2b^2 z A e^{-b(bz^2+vt)}$

$$\frac{\partial^2 f_4}{\partial z^2} = -2b^2 A e^{-b(bz^2+vt)}\left[1 - 2b^2 z^2\right]$$

$$\frac{1}{v^2}\frac{\partial f_4}{\partial t} = \frac{-bvA}{v^2} e^{-b(bz^2+vt)} = b^2 A e^{-b(bz^2+vt)}$$

) not = ✓

f_5: $f_5 = A \sin bz \cos(bvt)^3$ $\frac{\partial f_5}{\partial z} = Ab \cos bz \cos(bvt)^3$

$$\frac{\partial^2 f_5}{\partial z^2} = -Ab^2 \sin bz \cos(bvt)^3$$

$$\frac{1}{v^2}\frac{\partial f_5}{\partial t} = \frac{A}{v^2} \sin bz \; 3(bvt)^2 bv \left(-\sin(bvt)^3\right)$$

$$\frac{\partial^2 f_5}{\partial t^2} = -3Ab^3 v \sin bz \left[3b^3v^3t^2 \cos(bvt)^3 t^2 + 2t\sin(bvt)^3\right]$$

$$\frac{1}{v^2}\frac{\partial^2 f_5}{\partial t^2} = -6Ab^3 v t \sin bz \sin(bvt)^3 - 9Ab^6 v^4 t^4 \sin bz \cos(bvt)^3$$

not = ✓

Griffiths 9-2 $\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$ $f = A \sin kz \cos kvt$

$\frac{\partial f}{\partial z} = kA \cos kz \cos kvt$ $\frac{\partial^2 f}{\partial z^2} = -k^2 A \sin kz \cos kvt$

$\frac{1}{v^2}\frac{\partial f}{\partial t} = -\frac{Akv}{v^2} \sin kz \sin kvt$ $\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = -\frac{Ak}{v}(kv) \sin kz \cos kvt$

$= -Ak^2 \sin kz \cos kvt$ ✓

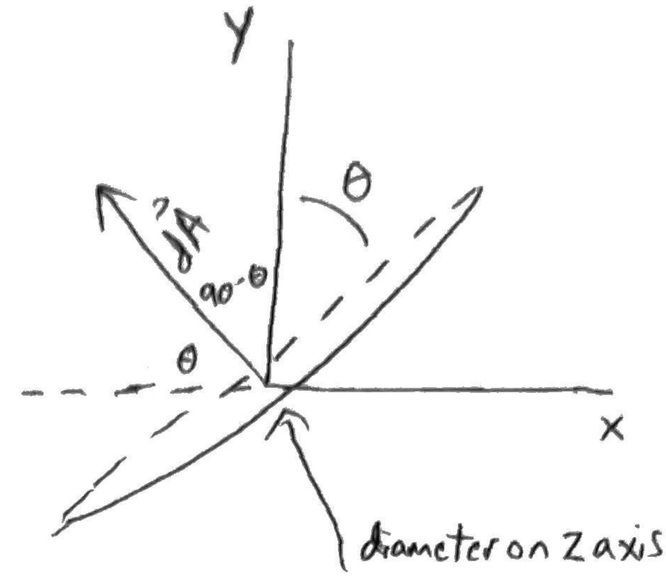
now $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

so $\sin(a+b) + \sin(a-b) = 2 \sin a \cos b$ w/ $a = kz$, $b = kvt$

so $\sin kz \cos kvt = \frac{1}{2}\left(\sin(kz-kvt) + \sin(kz+kvt)\right)$

or $f(z,t) = \frac{A}{2}\left[\sin(kz-kvt) + \sin(kz+kvt)\right]$

Wangsness 24-5



- $\vec{B} = \frac{k}{\omega} \hat{k} \times \vec{E}$ *

$= \frac{k}{\omega} \hat{z} \times E_0 e^{i(kz-\omega t)} \hat{y}$

so $\vec{B} = -\frac{k}{\omega} E_0 e^{i(kz-\omega t)} \hat{x}$

$\mathcal{E} = -\frac{d\Phi}{dt} = \frac{-d}{dt} \iint \vec{B} \cdot \vec{dA}$ now $a \ll \lambda$

so e^{ikz} could have as large an exponent as: $e^{i\frac{2\pi a}{\lambda}} \simeq e^0 \simeq 1$

- so $a \ll \lambda$ implies that $\vec{B}$ is constant over the loop

so $\iint \vec{B} \cdot \vec{dA} = |\vec{B}| \cos\theta \, A N$

$\mathcal{E} = \frac{-d}{dt}\left(\frac{k}{\omega} E_0 e^{-i\omega t} \cos\theta \, \pi a^2 N\right) = i k E_0 e^{-i\omega t} \cos\theta \, \pi a^2 N$

we want only the real part so $\underline{\mathcal{E} = k E_0 \pi a^2 N \cos\theta \sin\omega t}$

note: $\vec{dA}$ could certainly be reversed here since it isn't a closed surface. That would flip the sign of the answer.

- * we obtained this result in class. you should be able to prove it using Maxwell's equations

Wangsness 24-12

a) $\vec{E} = (\vec{E}_R + i\vec{E}_I)e^{-\beta z}e^{i(\alpha z-\omega t)} = e^{-\beta z}\Big[\vec{E}_R\cos(\alpha z-\omega t) + i\vec{E}_I\cos(\alpha z-\omega t) + i\vec{E}_R\sin(\alpha z-\omega t) - \vec{E}_I\sin(\alpha z-\omega t)\Big]$

where the actual E is just the real part

$w = \vec{J}_f\cdot\vec{E} = \sigma\vec{E}\cdot\vec{E}$

$= \sigma\,\mathrm{Re}(\vec{E})\cdot\mathrm{Re}(\vec{E})$

keeping only the real part of $\vec{E}$

so $\boxed{w = \sigma e^{-2\beta z}\Big[\vec{E}_R^2\cos^2(\alpha z-\omega t) + \vec{E}_I^2\sin^2(\alpha z-\omega t) - 2\vec{E}_I\cdot\vec{E}_R\cos(\alpha z-\omega t)\sin(\alpha z-\omega t)\Big]}$

$\langle w\rangle$: you could just time average $\langle\cos^2\theta\rangle = \langle\sin^2\theta\rangle = \frac{1}{2}$

$\langle\cos\theta\sin\theta\rangle = \emptyset$ $\Longrightarrow$ $\langle w\rangle = \frac{1}{2}\sigma e^{-2\beta z}\Big[\vec{E}_R^2 + \vec{E}_I^2\Big]$

or $\boxed{\langle w\rangle = \frac{1}{2}\sigma e^{-2\beta z}|\vec{E}_0|^2}$

or you could use $\langle w\rangle = \frac{1}{2}\mathrm{Re}(\sigma\vec{E}\cdot E^*)$ which will automatically be completely real in this case

$= \frac{1}{2}\mathrm{Re}\Big[\sigma(\vec{E}_R + i\vec{E}_I)e^{-\beta z}e^{i(\alpha z-\omega t)} * (\vec{E}_R - i\vec{E}_I)e^{-\beta z}e^{-i(\alpha z-\omega t)}\Big]$

$= \frac{1}{2}\mathrm{Re}\Big[\sigma e^{-2\beta z}(\vec{E}_R^2 + \vec{E}_I^2)\Big] \Longrightarrow \boxed{\langle w\rangle = \frac{1}{2}\sigma e^{-2\beta z}|\vec{E}_0|^2}$

b) $\frac{\text{power loss}}{\text{area}} = \int_0^\infty \langle w\rangle\, dz = \frac{1}{2}\sigma|\vec{E}_0|^2\int_0^\infty e^{-2\beta z}\, dz$

recall $\langle w\rangle$ - $\frac{\text{power loss}}{\text{volume}}$

$= \frac{1}{2}\sigma|\vec{E}_0|^2\frac{-1}{2\beta}(\emptyset - 1) \Longrightarrow \boxed{\frac{\sigma|\vec{E}_0|^2}{4\beta}}$

c) $\langle \vec{S} \rangle = \frac{1}{2} Re(\vec{E} \times H^*)$ $\quad \vec{B} = \frac{K}{\omega} \hat{z} \times \vec{E}$ (from class w/ $K = \alpha + iB$)

$H = B/\mu$ $\quad \cancel{\langle \vec{S} \rangle = Re\left[(\vec{E}_R + i\vec{E}_I) e^{-Bz} e^{i(\alpha z - \omega t)}\right] \times \left[\frac{K^*}{\mu\omega} \hat{z} \times\right]}$

$$\langle \vec{S} \rangle = \frac{1}{2} Re\left[\underbrace{\left((\vec{E}_R + i\vec{E}_I) e^{-Bz} e^{i(\alpha z - \omega t)}\right)}_{\vec{E}} \underbrace{\times}_{\times} \underbrace{\left(\frac{K^*}{\mu\omega} \hat{z} \times (\vec{E}_R - i\vec{E}_I) e^{-Bz} e^{-i(\alpha z - \omega t)}\right)}_{H^*}\right]$$

$$= \frac{1}{2} Re\left[\frac{e^{-2Bz}}{\mu\omega}(\alpha - iB)\left(\underbrace{\vec{E}_R \times (\hat{z} \times \vec{E}_R)}_{|\vec{E}_R|^2 \hat{z}} + \vec{E}_R \times (\hat{z} \times (-i\vec{E}_I)) + \vec{E}_I \times (\hat{z} \times \vec{E}_R) + \underbrace{\vec{E}_I \times (\hat{z} \times (-i)\vec{E}_I)}_{|\vec{E}_I|^2 \hat{z}}\right)\right]$$

cross terms cancel each other (this takes some thought...)

$$= \frac{1}{2} Re\left[\frac{e^{-2Bz}}{\mu\omega}(\alpha - iB)|\vec{E}_0|^2 \hat{z}\right]$$

$\frac{\alpha}{\omega} = \frac{1}{V}$ $\quad$ or $\quad \boxed{\langle \vec{S} \rangle = \frac{e^{-2Bz} |\vec{E}_0|^2 \hat{z}}{2\mu V}}$

$(2\alpha B = \omega\mu\sigma)$

d) $\langle \vec{S} \rangle \Big|_{z=\emptyset} = \frac{E_0^2 \hat{z}}{2\mu V} = \frac{E_0^2 \hat{z}\, \alpha}{2\omega\mu} = \frac{E_0^2 \hat{z}\, \sigma}{4B}$ ✓

in other words, the power per area passing $z = \emptyset$ is the same as the power per area lost by the time the wave reaches $z = \infty$

$\Rightarrow$ all energy entering the conductor is absorbed!

Wangsness 24-13

- let's get $\langle \vec{S} \rangle$ for each component 1st

$$\langle \vec{S} \rangle = \frac{1}{2}\mathrm{Re}(\vec{E}\times H^*) \quad \hat{H} = \frac{K}{\mu\omega}\hat{k}\times\vec{E} = \frac{1}{\mu v}\hat{k}\times\vec{E} = \sqrt{\frac{\epsilon}{\mu}}\,\hat{k}\times\vec{E}$$

$$\langle \vec{S} \rangle = \frac{1}{2}\mathrm{Re}\left(E_\alpha \hat{x}\, e^{i(\kappa z-\omega t+\theta_\alpha)} \times \left(\sqrt{\frac{\epsilon}{\mu}}\,\hat{z}\times E_\alpha \hat{x}\, e^{-i(\kappa z-\omega t+\theta_\alpha)}\right)\right)$$

$$= \frac{1}{2}\mathrm{Re}\left(E_\alpha^2 \sqrt{\frac{\epsilon}{\mu}}\,\hat{z}\right)$$

$$\boxed{\langle \vec{S}_\alpha \rangle = \frac{1}{2}E_\alpha^2\sqrt{\frac{\epsilon}{\mu}}\,\hat{z}}$$

and

$$\boxed{\langle \vec{S}_B \rangle = \frac{1}{2}E_B^2\sqrt{\frac{\epsilon}{\mu}}\,\hat{z}}$$

$$\langle \vec{S} \rangle = \frac{1}{2}\mathrm{Re}\left[\left(\hat{x}E_\alpha e^{i()} + \hat{y}E_B e^{i()}\right)\times\left(\hat{z}\times\left(\hat{x}E_\alpha e^{-i()} + \hat{y}E_B e^{-i()}\right)\right)\sqrt{\frac{\epsilon}{\mu}}\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\left(\hat{x}E_\alpha e^{i()} + \hat{y}E_B e^{i()}\right)\times\left(\hat{y}E_\alpha e^{-i()} + (-\hat{x})E_B e^{-i()}\right)\sqrt{\frac{\epsilon}{\mu}}\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\left(E_\alpha^2\hat{z} + E_B^2\hat{z}\right)\sqrt{\frac{\epsilon}{\mu}}\right]$$

$$= \boxed{\frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}E_\alpha^2\hat{z} + \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}E_B^2\hat{z} = \langle \vec{S}_\alpha \rangle + \langle \vec{S}_B \rangle}$$

since $\vec{E}_\alpha \cdot \vec{E}_B = \emptyset$ there is no interference between the two. <u>In a more general situation</u>,

$$\langle \vec{S}_{total} \rangle \neq \langle \vec{S}_\alpha \rangle + \langle \vec{S}_B \rangle$$

Griffiths 9-18

a) $\vec{\nabla}\cdot\vec{J}_f = -\frac{\partial \rho_f}{\partial t}$ $\quad \vec{J}_f = \sigma\vec{E}$ so $\sigma\vec{\nabla}\cdot\vec{E} = -\frac{\partial \rho_f}{\partial t}$ $\quad \sigma\frac{\rho_f}{\epsilon} = -\frac{\partial \rho_f}{\partial t}$

or $\rho_f(t) = \rho_f(t=\not{0})\, e^{-\sigma t/\epsilon}$ so ϵ/σ is a time constant for the decay of the charge. (compare to $\lambda_f = \lambda_{f0} e^{-\sigma t/\epsilon}$)
from 21-2 which we solved in class

So ϵ/σ __approximates!__ the time required to reach the surface. $\sigma = 10^{-12}\,\Omega^{-1}m^{-1}$

ϵ: $\quad n = \frac{c}{v} = \frac{\sqrt{\mu\epsilon}}{\sqrt{\mu_0\epsilon_0}}$ $\quad \mu_0\epsilon_0 n^2 = \mu\epsilon$ so $\epsilon = \epsilon_0 n^2$ for $\mu = \mu_0$

$$\tau = \epsilon/\sigma = \frac{1.5^2 \times 8.85\times10^{-12}}{10^{-12}} \simeq \underline{20s}$$

reminder: this is an estimate and σ_{glass} could easily vary by a factor of 100

b) $\delta = 1/\beta$. you should make the silver just a bit deeper than the skin depth. for a good conductor $(\sigma \to \infty)$ like silver $Q = \frac{\omega\epsilon}{\sigma}$

$$\alpha = \beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left(\frac{1}{Q^2}\right)^{1/4} = \omega\sqrt{\frac{\mu\epsilon}{2}}\frac{\sqrt{\sigma}}{\sqrt{\omega\epsilon}} \qquad \beta = \sqrt{\frac{\omega\sigma\mu}{2}}$$

so $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$

$$= \sqrt{\frac{2}{2\pi\times10^{10}\times 4\pi\times10^{-7}\times 6.29\times10^{7}}} = 6.35\times10^{-7}\,m$$

$= 6.35\times10^{-4}\,mm$ $\quad$ so $.001\,mm$ of A_g should be enough

c) again in a good conductor $\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$ $\lambda = 2\pi/\alpha$

so $\lambda = 2\pi\sqrt{\frac{2}{\omega\mu\sigma}} = 2\pi\sqrt{\frac{2}{2\pi\times10^{6}\times 4\pi\times10^{-7}\times 5.95\times10^{7}}}$

$$\boxed{\lambda = .4\,mm}$$

in vacuum $\lambda = \frac{3\times10^{8}}{10^{6}} \approx \underline{300\,m\,!}$

the speed is reduced by the same factor

so $V = 3\times10^{8}\left(\frac{.4\,mm}{300\,m}\right) = \underline{400\,m/s\,!}$

note: as part b suggests, the skin depth in a good conductor is ~~■~~ $<\lambda$ * so the notion of a wavelength or speed becomes a bit vague...

* see next problem

Griffiths 9-19

$Q = \omega\epsilon/\sigma$

a) $\alpha = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+1/Q^2}+1\right]^{1/2}$ $\quad B = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+1/Q^2}-1\right]^{1/2}$

$\delta = 1/B \quad B = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\underbrace{1+\frac{1}{2}\frac{\sigma^2}{\epsilon^2\omega^2}+\cdots}_{\text{binomiol expansion}}-1\right]^{1/2}$

$B = \omega\sqrt{\frac{\epsilon\mu}{2}}\frac{1}{\sqrt{2}}\frac{\sigma}{\epsilon\omega}$ or $B = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$ or $\boxed{\delta = \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}}$

poor conductor

$\delta_{water} = \frac{2}{4\times10^{-6}}\sqrt{\frac{80.1\times8.85\times10^{-12}}{4\pi\times10^{-7}}}$

$\Rightarrow \boxed{\delta_{water} = 11.9\ Km}$ - again this is a poor conductor it loses energy gradually as it propogates

b) from the previous problem

$\delta = \sqrt{2/\omega\mu\sigma}$ and $\lambda = 2\pi\sqrt{2/\mu\omega\sigma}$ so clearly $\boxed{\delta = \lambda/2\pi}$

rest of b on other side

c) we already showed that $\alpha = B$ for a good conductor

$\tilde{E} = \tilde{E}_0 e^{-Bz} e^{i(\alpha z-\omega t)}$ $\quad \tilde{B} = \frac{|k|}{\omega} e^{i\Omega}\,\hat{z}\times\tilde{E}_0 e^{-Bz} e^{i(\alpha z-\omega t)}$ $\quad \tan\Omega = \frac{B}{\alpha}$

so $\boxed{\Omega = 45^0}$

so $\frac{|\tilde{B}|}{|\tilde{E}|} = \frac{|k|}{\omega}$ from previous problem

$\alpha = B = \sqrt{\frac{\omega\sigma\mu}{2}}$ $\quad |k| = \sqrt{\alpha^2+B^2} = \sqrt{\omega\sigma\mu}$

so $\frac{|\tilde{B}|}{|\tilde{E}|} = \sqrt{\frac{\sigma\mu}{\omega}} = \sqrt{\frac{10^7\,4\pi\times10^{-7}}{10^{15}}} = \underline{1.12\times10^{-7}\ s/m}$

[redacted]

in a vacuum $B/E = 1/c = 3.33 \times 10^{-9}$ s/m

so $\vec{B}$ is comparatively larger in the conductor

back to b: $\delta = \sqrt{\dfrac{2}{\omega \sigma \mu}} = \sqrt{\dfrac{2}{10^{15} \times 10^{7} \times 4\pi \times 10^{-7}}}$

$\Rightarrow \delta = 12.6$ nm - energy absorbed in a very short distance

so metals are opaque since visible light that tries to penetrate the metal is absorbed in an extremely short distance

Griffiths 9-20 part a

since we are given the real fields, we will be calculating the time average directly (and not using $\frac{1}{2}Re(\vec{D}\cdot\vec{E}^*)+\frac{1}{2}Re(\vec{B}\cdot H^*)$)

$$u = \frac{1}{2}\vec{D}\cdot\vec{E} + \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2 = \frac{1}{2}\epsilon E^2 + \frac{B^2}{2\mu}$$

$$= \frac{1}{2}\epsilon E_0^2 e^{-2Bz}\cos^2(\alpha z - \omega t + \delta_E) + \frac{1}{2\mu}B_0^2 e^{-2Bz}\cos^2(\alpha z - \omega t + \delta_E + \Omega)$$

$$\langle u\rangle = \frac{1}{2}e^{-2Bz}\left(E_0^2\epsilon\langle\cos^2\theta\rangle + \frac{B_0^2}{\mu}\langle\cos^2\theta\rangle\right) \qquad \langle\cos^2\theta\rangle = 1/2$$

now $\vec{B} = \frac{|K|}{\omega}\hat{k}\times\vec{E}$ so $B_0^2 = K^2/\omega^2 E_0^2$ $\quad K^2 = \alpha^2 + B^2$

$$\begin{matrix}\alpha \\ B\end{matrix} = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+1/Q^2} \pm 1\right]^{1/2} \qquad K^2 = \omega^2\frac{\mu\epsilon}{2}\left[\sqrt{\ } + 1 + \sqrt{\ } - 1\right]$$

$$K^2 = \omega^2\mu\epsilon\sqrt{1+1/Q^2} \qquad \langle u\rangle = \frac{1}{4}e^{-2Bz}\left(\epsilon E_0^2 + \frac{1}{\mu}\frac{\omega^2\mu\epsilon\sqrt{1+1/Q^2}}{\omega^2}E_0^2\right)$$

$$\boxed{\langle u\rangle = \frac{1}{4}e^{-2Bz}\epsilon E_0^2\left(1+\sqrt{1+1/Q^2}\right) \qquad Q = \omega\epsilon/\sigma} \quad ①$$

or since $1+\sqrt{\ } = \frac{2\alpha^2}{\omega^2\mu\epsilon}$ $\quad \boxed{\langle u\rangle = \frac{\alpha^2 E_0^2 e^{-2Bz}}{2\mu\omega^2}}$

① above shows that $\frac{\langle u_B\rangle}{\langle u_E\rangle} = \frac{\sqrt{1+1/Q^2}}{1} = \sqrt{1+\sigma^2/\epsilon^2\omega^2}$

so as long as $\sigma \neq 0$, $\langle u_B\rangle > \langle u_E\rangle$.

For a good conductor, the magnetic contribution dominates

Griffiths 9-12 $\quad T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij} E^2 \right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2 \right)$

$\vec{E}$ has only an x-comp and $\vec{B}$ has only a $\hat{y}$-comp

so $T_{xy} = T_{yx} = T_{xz} = T_{zx} = T_{yz} = T_{zy} = \phi$ $\qquad E = E_0 \cos(\;)$

$\qquad B = \frac{E_0}{c}\cos(\;)$

$$T_{xx} = \epsilon_0 \left(E^2 - \frac{1}{2}E^2 \right) + \frac{1}{\mu_0}\left(\phi - \frac{1}{2}B^2 \right)$$

$$= \frac{1}{2}\epsilon_0 E_0^2 \cos^2(\;) - \frac{1}{2\mu_0} E_0^2 \mu_0 \epsilon_0 \cos^2(\;) = \phi$$

$$T_{yy} = \epsilon_0 \left(\phi - \frac{1}{2}E^2 \right) + \frac{1}{\mu_0}\left(B^2 - \frac{1}{2}B^2 \right)$$

$$= -\frac{1}{2}\epsilon_0 E_0^2 \cos^2(\;) + \frac{1}{2\mu_0} E_0^2 \mu_0 \epsilon_0 \cos^2(\;) = \phi$$

$$T_{zz} = \epsilon_0 \left(\phi - \frac{1}{2}E^2 \right) + \frac{1}{\mu_0}\left(\phi - \frac{1}{2}B^2 \right) = -\frac{1}{2}\epsilon_0 E^2 - \frac{1}{2}\frac{B^2}{\mu_0}$$

(which is just $-$ energy density!)

$$\epsilon_0 E^2 = B^2/\mu_0$$

so $\boxed{T_{zz} = -u = -\epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)}$ all other components are ϕ

Since the wave is traveling in z, it makes sense that it can exert forces in z and/or transfer momentum in that direction.

now for light $E = pc$ so $u = gc$ $\qquad gc$ is now $\frac{\text{momentum}}{\text{area time}}$

so it's the same (to within signs) as T_{zz}

so the $T_{zz} = -u$ makes sense.

[redacted]

Griffiths 9-8

A is real

a) $\tilde{f}(z,t) = A\,\cancel{\cos}\,e^{i(kz-\omega t)}\,\hat{x} + A\,e^{i(kz-\omega t+\pi/2)}\,\hat{y}$ the real part is the actual vector

imagine that you are at a z such that $0 < kz < \pi/2$

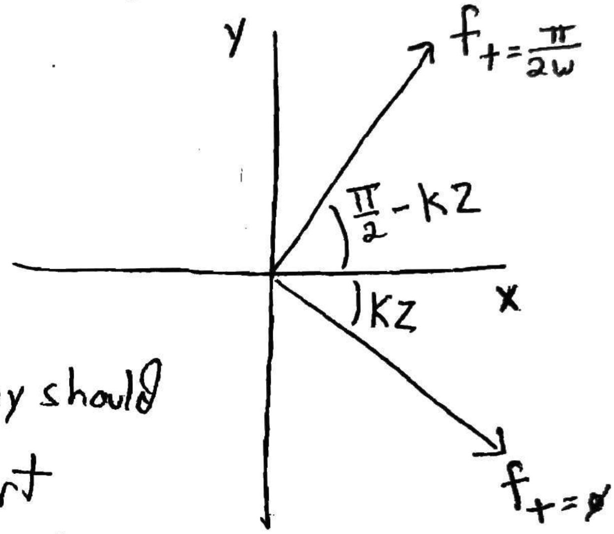
Graph $\tilde{f}_{t=0}$ and $\tilde{f}_{t=\frac{\pi}{2\omega}}$

$\tilde{f}_{t=0} = A\cos kz\,\hat{x} + A\cos\left(kz+\frac{\pi}{2}\right)\hat{y}$

$\tilde{f}_{t=\frac{\pi}{2\omega}} = A\cos\left(kz-\frac{\pi}{2}\right)\hat{x} + A\cos kz\,\hat{y}$

so $\underline{\tilde{f}_{t=0} = A\cos kz\,\hat{x} - A\sin kz\,\hat{y}}$

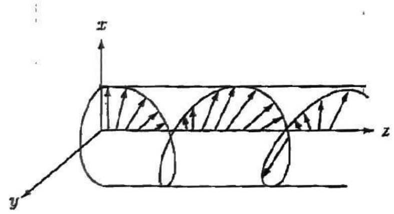
so $\underline{\tilde{f}_{t=\frac{\pi}{2\omega}} = A\sin kz\,\hat{x} + A\cos kz\,\hat{y}}$



so it rotates <u>CCW</u>

note that $\tilde{f}_{t=0}\cdot\tilde{f}_{t=\frac{\pi}{2\omega}} = 0$ as they should since they are 1/4 of a cycle apart

b)



c) rotate the end of the string in a circle