PHYSICS 515-A

ELECTROMAGNETIC THEORY

Section IV(b) Problems (due May 6^{th})

Problem 1: Consider a single accelerated charge q.

(a) Show that the power emitted into solid angle $d\Omega$ can be written

$$\frac{dP}{d\Omega} = \frac{q^2\dot{\beta}^2}{4\pi c}\sin^2\theta \ ,$$

where $v = \beta c$ is the particle's velocity. Define θ.

(b) Sketch the radiation pattern.

(c) Obtain the total emitted power (Larmor's formula).

Problem 2: Suppose an electromagnetic wave $\vec{E} = E_0\vec{\epsilon}\sin\omega_0 t$ is incident on a free charge q.

(a) What is the amplitude of the resulting oscillating dipole $\vec{p}$?

(b) Show that in the radiation zone, the instantaneous power emitted into solid angle $d\Omega$ is

$$\frac{dP}{d\Omega} = \frac{\sin^2\theta}{4\pi c^3}\left(\frac{d^2\vec{p}}{dt^2}\right)^2 \ .$$

Define θ.

(c) Defining the *Differential cross section* $d\sigma$ for scattering into $d\Omega$ by the relation

$$\langle S\rangle\frac{d\sigma}{d\Omega} = \frac{dP}{d\Omega} \ ,$$

where $\langle S\rangle$ is the time-averaged incident flux, show that the total electron cross section σ equals the Thomson cross section

$$\sigma_T \equiv \frac{8\pi}{3}r_0^2 \ ,$$

where $r_0 \equiv e^2/m_e c^2$ is the classical electron radius.

Problem 3: A pulsar is conventionally believed to be a rotating neutron star. Such a star is likely to have a strong magnetic field, B_0, since it traps lines of force during its collapse. If the magnetic axis of the neutron star does not line up with the rotation axis, there will be magnetic dipole radiation from the time-changing magnetic dipole, $\vec{\mu}(t)$. Assume that the mass and radius of the neutron star are M and R, respectively; that the angle between the magnetic and rotation axes is α; and that the rotational angular velocity is ω.

(a) Find an expression for the radiated power P in terms of ω, R, B_0, and α.

(b) Assuming that the rotational energy of the pulsar is the ultimate source of the radiated power, find an expression for the slow-down time scale, $\tau \equiv -\omega/\dot{\omega}$, of the pulsar.

(c) For $M = 2 \times 10^{33}$ g, $R = 10^6$ cm, $B_0 = 10^{12}$ gauss, $\alpha = 90^o$, find P and τ for $\omega = 10^4$ s^{-1}; 10^2 s^{-1}. (The larger of these two rates is believed to be typical of newly formed pulsars.)

Problem 4: A particle of mass m and charge q moves at constant, nonrelativistic speed $v_\perp$ in a circle of radius a.

(a) What is the power emitted per unit solid angle in a direction at angle θ to the axis of the circle?

(b) Describe qualitatively and quantitatively the polarization of the radiation as a function of the angle θ.

(c) What is the spectrum of the emitted radiation?

(d) Suppose a particle is moving nonrelativistically in a constant magnetic field B. Show that the frequency of circular motion is $\omega_B = eB/mc$, and that the total emitted power is

$$P = \frac{2}{3} r_0^2 c (v_\perp/c)^2 B^2 \ ,$$

and is emitted solely at the frequency ω_B. (This nonrelativistic form of synchrotron radiation is called *cyclotron* or *gyro* radiation.)

(e) Find the differential and total cross section for Thomson scattering of circularly polarized radiation. Use these results to find the cross sections for unpolarized radiation.