PHYSICS 515B

ELECTROMAGNETIC THEORY

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Section V Problems Part II (due Wednesday, October 9)

Problem 1: (a) For the reaction $ab \to 12$, we may define the following Lorentz invariant variables

$$s = (p_a + p_b)^2$$

$$t = (p_a - p_1)^2$$

$$u = (p_a - p_2)^2$$

where $p^2 \equiv p^\alpha p_\alpha$. Show that only two of these variables are independent by finding the constant (C) in the equation

$$s + t + u = C\ .$$

(b) Relate t and u to $\cos\theta^*$, the angle in the CMS (center of momentum frame) between the incident particle a and the produced particle 1. Express your answer in terms of the kinematical function

$$\lambda(x, y, z) \equiv (x - y - z)^2 - 4yz.$$

Find the values of $\cos\theta^*$ which correspond to the maximum and minimum allowed values of t and u.

Problem 2: Consider the elastic collision of a projectile, of momentum $\vec{p}_1$ and energy E_1, with a stationary target of mass m_2. Show that, for the scattering of m_2 through the CMS angle θ^*, the final *lab* kinetic energy of the target particle is

$$K_2 = \frac{2m_2 p_1^2 c^4}{E_0^2} \sin^2\frac{\theta^*}{2}\ ,$$

where $E_0^2 = sc^2$ is the invariant energy squared.

Problem 3: It is sometimes useful to understand Lorentz transformations of quantities in terms of a piecemeal intuitive approach, as well as in terms of the elegant language of tensor transformations. For example, by means of a simple physical model we can derive the transformation of the electromagnetic fields $\vec{E}$ and $\vec{B}$ for the case of an initially pure electric field $\vec{B} = 0$.

(a) Consider a charged capacitor with plates perpendicular to the z axis in its own rest frame K. Let σ be the surface charge density. Then it is known that the electric field inside is $E = 4\pi\sigma$, independent of the separation of the plates d and has a direction normal to the plates. Discuss the properties of the capacitor as seen in a reference frame K' moving with velocity v along the z-axis relative to K (i.e., what is σ', d', etc.). Show that

$$\vec{E}'_{\|} = \vec{E}_{\|}.$$

(b) Now consider the capacitor turned so that the plates are perpendicular to the y-axis. Again discuss the properties of the capacitor as seen in K' and thereby show that

$$\vec{E}'_{\perp} = \gamma \vec{E}_{\perp} \ ,$$

and

$$\vec{B}'_{\perp} = -\gamma \vec{\beta} \times \vec{E}_{\perp} \ .$$

Problem 4: Jackson 11.16.

Problem 5: Consider the motion of a point charge in crossed uniform fields $\vec{E} \perp \vec{B}$.
(a) Show that the fields in the frame *drifting* with velocity $\vec{v} = c\vec{E} \times \vec{B}/B^2$ are

$$\vec{E}' = 0 \ ,$$

$$\vec{B}' = \vec{B} \left(1 - \frac{E^2}{B^2} \right)^{1/2} \ ,$$

What is the significance of this frame?
(b) Show that in this drifting frame, the particle undergoes circular motion with angular frequency

$$\omega' = \frac{u'_{\perp}}{R'} \ ,$$

where R' is the radius of the circle and $u'_{\perp}$ is the component perpendicular to the magnetic field of the particle's velocity relative to the drifting frame. Find R' and ω'.
(c) Discuss the relativistic distortions to the non-relativisitc cycloidal motion, for the special case of a particle starting from rest in the laboratory, with particular attention to the circulation radius (and therefore the cycle length).