PHYSICS 515B
ELECTROMAGNETIC THEORY
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Fall 2025

Homework 5 (due Wednesday, December 10)

Problem 1: In lecture, we found that the potential from a point charge could be written

$$A^\mu(x) = 2q\int d^4x'\ \theta(x_0 - x_0')\delta[(x-x')^2]\int d\tau\ U^\mu(\tau)\delta^{(4)}(x' - x_q(\tau))\ .$$

Show that in terms of the scalar potential Φ and vector potential $\vec{A}$, this reduces to

$$\Phi(\vec{x},t) = \left[\frac{q}{(1-\vec{\beta}\cdot\hat{n})|\vec{x}-\vec{x}_q|)}\right]_{ret}\ ,$$

and

$$\vec{A}(\vec{x},t) = \left[\frac{q\vec{\beta}}{(1-\vec{\beta}\cdot\hat{n})|\vec{x}-\vec{x}_q|}\right]_{ret}\ ,$$

where "ret" means the quantity in the brackets is to be evaluated at the retared time τ_0, such that

$$(x_0 - x_q(\tau_0))^2 = |\vec{x}-\vec{x}_q(\tau_0)|^2\ .$$

Problem 2: In this problem, we will use relativistic transformations to find the radiation emitted by a particle moving at relativistic speeds.

(a) Show that the total emitted power is a Lorentz invariant for any emitter that emits with front-back symmetry in its instantaneous rest frame.

(b) Show that the covariant generalization of the Larmor formula is

$$P = \frac{2q^2}{3c^3}a^\alpha a_\alpha\ ,$$

where $a^\alpha \equiv dU^\alpha/d\tau$ and U^α is the 4-velocity. Be careful about which frame you're in!

(c) Show that in terms of the 3-vector acceleration $d^2\vec{x}/dt^2$, this power is

$$P = \frac{2q^2}{3c^3}\gamma^4(a_\perp^2 + \gamma^2 a_\parallel^2)\ ,$$

where $a_\parallel$ and $a_\perp$ are the components of acceleration parallel and perpendicular to the direction of v.

Problem 3: Suppose a particle of mass m and charge q is moving (relativistically) in a uniform magnetic field $\vec{B}$.

(a) Show that the total emitted power may be written

$$P = 2\sigma_T c \beta^2 \gamma^2 U_B \ ,$$

where σ_T is the Thomson cross section, $\beta = v/c$, and U_B is the magnetic energy density.

(b) Assuming an observer sees the radiation coming only within the cone of half-angle $1/\gamma$ about the velocity vector, show that the duration of the *observed* pulse once every gyration period is

$$\Delta t \approx \frac{2}{\gamma \omega_B \sin\alpha} \left(1 - \frac{v}{c}\right) \ ,$$

where ω_B is the angular velocity of gyration, and α is the pitch angle, i.e., the angle between the velocity vector and the magnetic field.

Problem 4: A particle is accelerated by a force having components $F_\parallel$ and $F_\perp$ with respect to the particle's velocity. Show that the radiated power is

$$P = \left(\frac{2q^2}{3m^2c^3}\right)(F_\parallel^2 + \gamma^2 F_\perp^2) \ .$$

Thus, the perpendicular component has more effect in producing radiation than the parallel component by a factor γ^2.

Problem 5: An object emits a blob of material at speed v at an angle θ to the line-of-sight of a distant observer.

(a) Show that the apparent transverse velocity inferred by the observer is

$$v_{app} = \frac{v \sin\theta}{1 - (v/c)\cos\theta} \ .$$

(b) Show that v_{app} can exceed c; find the angle for which v_{app} is maximum, and show that this maximum is $v_{max} = \gamma v$.