

**Phys 332**  
**Electricity and Magnetism II**  
 Prof. Fulvio Melia  
 Homework 1

**Problem 1:** Wangsness 21-1.

**Problem 2:** Wangsness 21-7.

**Problem 3:** Wangsness 21-9.

**Problem 4:** Wangsness 21-10.

**Problem 5:**

(a) A very long solenoid of radius  $a$  and  $n$  turns per unit length carries a current  $I_s$ . Coaxial with the solenoid, at radius  $b \gg a$  is a circular ring of wire with resistance  $R$ . When the current in the solenoid is gradually decreased, a current  $I_r$  is induced in the ring. Determine  $I_r$  in terms of  $dI_s/dt$ . You should assume that any displacement currents are negligible.

(b) The power  $I_r^2 R$  delivered to the ring must have come from the solenoid. Confirm this by calculating the Poynting vector at  $\rho = a$ . Integrate this Poynting vector over the surface of the solenoid and verify that you recover the correct total power. (Note that the electric field is due to the changing flux in the solenoid while the magnetic field is due to the current in the ring. Since  $b \gg a$ , you can assume that you just need the magnetic field on the axis of a ring of current.)

**Problem 6:** Consider two equal but opposite point charges separated by a distance  $2a$ . Construct the infinite plane equidistant from the two charges and the infinite hemisphere that surrounds one of the charges. Integrate the Maxwell stress tensor over this infinite surface and effectively verify Coulomb's Law.

**Problem 7:** Consider an infinite parallel-plate capacitor, with the lower plate (at  $z = -d/2$ ) carrying a charge density  $-\sigma$ , and the upper plate (at  $z = +d/2$ ) carrying a charge density  $+\sigma$ .

(a) Determine all nine elements of the stress tensor in the region between the plates. Display your answer as a  $3 \times 3$  matrix:

$$\begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}. \quad (1)$$

(b) Determine the force per unit area on the top plate.

(c) What is the momentum per unit area, per unit time, crossing the  $xy$  plane (or any other plane parallel to that one, between the plates)?

(d) At the plates, this momentum is absorbed and the plates recoil (unless there is some nonelectrical force holding them in position). Find the recoil force per unit area on the top plate, and compare your answer to (b). (Note that this is not an additional force but, rather, an alternative way of calculating the same force.)