**Phys 332**
**Electricity and Magnetism II**
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Homework 2

**Problem 1:** A charged parallel-plate capacitor (effectively infinite but with plate area $A$ and plate separation $d$) is placed in a uniform magnetic field $\mathbf{B} = B\hat{x}$. The plates are parallel to the $xy$ plane so that the uniform electric field between the plates is $\mathbf{E} = E\hat{z}$.

(a) Find the electromagnetic momentum in the space between the plates.

(b) Now a resistive wire is connected between the plates, along the $z$-axis, so that the capacitor slowly discharges. The current through the wire will experience a magnetic force. What is the total impulse delivered to the system during the discharge?

**Problem 2:** Imagine an iron sphere of radius $R$ that carries a charge $Q$ and a uniform magnetization $\mathbf{M} = M\hat{z}$ (so that $\mathbf{m} = 4\pi R^3\mathbf{M}/3$). The sphere is initially at rest. Compute the angular momentum stored in the electromagnetic fields. Instead of deriving $\mathbf{B}$, just use

$$\begin{aligned}\mathbf{B} &= \frac{2}{3}\mu_0 M\hat{z} \qquad (z < R)\\ &= \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta}) \qquad (z \geq R)\ .\end{aligned}$$

**Problem 3:**

(a) Suppose $\phi = 0$ and $\mathbf{A} = A_0\sin(kx - \omega t)\hat{y}$, where $A_0$, $\omega$, and $k$ are constants. Find $\mathbf{E}$ and $\mathbf{B}$, and check that they satisfy Maxwell's equations in vacuum. What conditions must you impose on $\omega$ and $k$?

(b) Also, determine $\mathbf{S}$.

**Problem 4:** Which of the following potentials are in the Coulomb gauge? Which are in the Lorentz gauge? Note that these gauges are not mutually exclusive. You may assume that $\sigma = 0$.

$$\begin{aligned}\phi &= 0 \qquad \mathbf{A} = \frac{\mu_0 k}{4c}(ct - |x|)^2\hat{z} \text{ for } |x| < ct, \text{ and } 0 \text{ for } |x| > ct\ ,\\ \phi &= 0 \qquad \mathbf{A} = -\frac{qt\hat{r}}{4\pi\epsilon_0 r^2}\ ,\\ \phi &= 0 \qquad \mathbf{A} = A_0\sin(kx - \omega t)\hat{y}\ .\end{aligned}$$

**Problem 5:** It is always possible to pick a vector potential whose divergence is zero (Coulomb gauge). Show that it is always possible to choose

$$\vec{\nabla}\cdot\mathbf{A} = -\mu_0\epsilon_0\frac{\partial\phi}{\partial t}\ , \qquad (1)$$

as required for the Lorentz gauge, assuming you know how to solve equations of the form $\Box^2\phi = -\rho/\epsilon_0$ or $\Box^2\mathbf{A} = -\mu_0\mathbf{J}$. Is it always possible to pick $\phi = 0$? How about $\mathbf{A} = 0$? Again, you may assume that $\sigma = 0$.

**Problem 6:** Wangsness 22-3.