Phys 332
Electricity and Magnetism II
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Homework 4

Problem 1:

(a) At normal incidence (on a surface in the $z = 0$ plane with $\sigma_f = 0$ and $\mathbf{K}_f = 0$), we effectively assumed that the plane of polarization of **E** did not change. Let's relax that assumption. Assume

$$\mathbf{E} = E_0 e^{i(kz-\omega t)}\hat{x}$$
$$\mathbf{E}'' = E_0'' e^{-i(kz+\omega t)}(\cos\theta\hat{x} + \sin\theta\hat{y})$$
$$\mathbf{E}' = E_0' e^{i(kz-\omega t)}(\cos\phi\hat{x} + \sin\phi\hat{y})$$

Prove that $\phi = \theta = 0$ by applying appropriate boundary conditions on the tangential components of **E** and **H**.

(b) Determine the ratio(s) (n/n') for which the reflected and transmitted intensities are each $1/2$. You should still assume normal incidence—and also assume $\mu = \mu'$.

Problem 2: Let's verify that energy is conserved for the general solution to the problem with **E** perpendicular to the plane of incidence. One way to do this is to compare the incident power per unit area to the reflected power per unit area plus the trnasmitted power per unit area. In other words,

$$|\langle\mathbf{s}\rangle\cdot\hat{n}| = |\langle\mathbf{s}'\rangle\cdot\hat{n}| + |\langle\mathbf{s}''\rangle\cdot\hat{n}| \ .$$

So use the electric field amplitude ratios from class and check that energy is conserved. Also assume that $\mu = \mu'$.

Problem 3: Wangsness 25-3.

Problem 4:

(a) Determine the ratio E_0'/E_0 at the Brewster angle assuming that **E** is parallel to the plane of incidence and that $\mu = \mu'$.

(b) Although the electric field amplitudes are different, verify that the wave energy densities are the same.

(c) Are the magnitudes and directions of the Poynting vectors the same? Explain/calculate.

Problem 5:

(a) A wave is incident normally on an ideal conductor ($\mathbf{E} = \mathbf{H} = 0$), whose surface is the $z = 0$ plane. Use $\mathbf{E} = E_0 e^{i(iz-\omega t)}\hat{x}$. Determine the total electric field in the region $z < 0$. Show that it is a standing wave and graph E_x as a function of z for $z < 0$ at $\omega t = \pi/2$ and at $\omega t = 3\pi/2$.

(b) Determine σ_f on the conducting surface.

(c) Determine $\mathbf{K}_f$ on the conducting surface. Does this $\mathbf{K}_f$ produce **H** in the right direction?