Phys 332

Electricity and Magnetism II

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Homework 5

PHYS332 Homework - Retarded Potentials and Radiation

1 : Griffiths 10-9

a: Suppose that an infinite wire carries a linearly increasing current $I(t) = kt$, for $t > 0$. Find the electric and magnetic fields generated.

b: Do the same for the case of a sudden burst of current: $I(t) = q_0\delta(t)$.

In part **b**, the integral would originally be over z. You will need to switch that to an integral over R otherwise you cannot easily use the delta function. Lastly, you will also need to know that ($\delta(kx) = \frac{1}{|k|}\delta(x)$) to actually evaluate the integral.

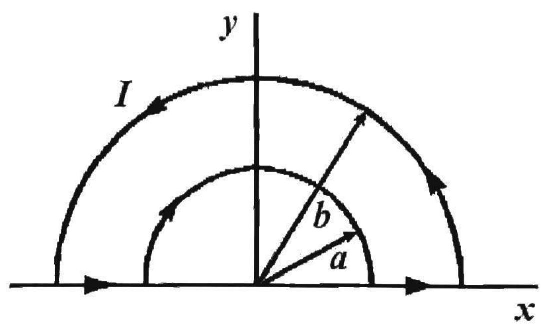
Note that only two points on the wire end up contributing to $\vec{A}$ since the current was present for only an instant and a particular point can only contribute if its signal has reached the field point at the appropriate time in question.

2 : Retarded Potentials - 1

Determine $\vec{A}$ a distance z above the xy plane if an infinite surface current $\vec{K} = \alpha t\hat{x}$ suddenly begins flowing in the xy plane at $t = 0$.

3 : Griffiths 10-10

A piece of wire bent into a loop, as shown in the figure, carries a current that increases linearly with time: $I(t) = kt$.



Calculate the retarded vector potential $\vec{A}$ at the center. Find the electric field at the center. Why does this *neutral* wire produce an *electric* field? (Why can't you determine the *magnetic* field from this expression for $\vec{A}$?)

4 : Wangsness 28-4

Find the fraction of energy radiated by an electric dipole within $\pm 10^0$ of the equatorial plane.

5 : Wangsness 28-5

Assume that p_0 is real and find the general physical fields $\vec{E}$ and $\vec{B}$ for an electric dipole from the expressions for $\vec{E}_{ed}$ and $\vec{B}_{ed}$ from class. Then find $\vec{S}$ and show that it has oscillatory components along $\hat{r}$ and $\hat{\theta}$. Lastly, find $< \vec{S} >$ and show that it agrees with the result from class obtained for the radiation zone.

6 : Wangsness 28-6

Show that the fields in the radiation zone due to an electric dipole can be written as:

$$\vec{E} = \frac{\mu_0}{4\pi r}\left(\left[\frac{d^2\vec{p}}{dt^2}\right] \times \hat{r}\right) \times \hat{r}$$

$$\vec{B} = \frac{\mu_0}{4\pi c r}\left[\frac{d^2\vec{p}}{dt^2}\right] \times \hat{r}$$

where $[x]$ implies that x is evaluated at the retarded time.

7 : Griffiths 11-10

An electron is released from rest on the Earth and falls under the influence of gravity. In the first centimeter, what fraction of the change in gravitational potential energy is converted to radiation?

Also, repeat this for an object falling near the surface of a neutron star with $M = 1.4M_{\odot} = 2.8 \times 10^{30}$ kg and $R = 10$ km. You may assume that Keplerian gravity is an acceptable approximation (it's probably off by about 30%).

8 : Radiation - 1

A proton is undergoing cyclotron motion in a magnetic field near the surface of a neutron star. Assume that $v_{proton} = 0.01c$, $|\vec{B}| = 10^8$ T and that $\vec{v} \perp \vec{B}$. Determine the power radiated by the proton. Compare this power to the proton's initial kinetic energy to *estimate* the time for the orbit to decay. Then do an appropriate integral to determine the time for the orbit to decay to one-tenth of the initial radius. Why can't you determine the time for the orbit to completely decay? Was it acceptable to ignore special relativity in this problem?