PHYS 332 Homework - Relativity and Relativistic Kinematics

1: Griffiths 12.4
As the outlaws escape in the getaway car which travels at $\frac{3}{4}c$, the police officer fires a bullet from the pursuit car which travels at only $\frac{1}{2}c$. The muzzle velocity of the bullet (relative to the gun) is $\frac{1}{3}c$. Does the bullet reach its target according to Galileo or according to Einstein?

2: Wangsness 29.4
A rigid rod of length L makes an angle θ with the x axis of the system in which it is at rest. Show that, for an observer moving along the x axis with respect to the rod, the apparent length L' and angle θ' are given by $L' = L\left[(\cos\theta/\gamma)^2 + \sin^2\theta\right]^{1/2}$, $\tan\theta' = \gamma\tan\theta$.

3: Marion/Thornton 14.12
A racer attempting to break the land speed record rockets by two markers spaced 100 m apart on the ground in a time of 0.4 μs as measured by an observer on the ground. How far apart do the two markers appear to the racer? What elapsed time does the racer measure? What speeds do the racer and ground observer measure? Explain.

4: Griffiths 12.18

a: Write out the matrix that describes a Galilean transformation.

$$\begin{array}{ll} \text{(i)} & \bar{x} = x - vt \\ \text{(ii)} & \bar{y} = y \\ \text{(iii)} & \bar{z} = z \\ \text{(iv)} & \bar{t} = t \end{array}$$

b: Write out the matrix describing a Lorentz transformation along the y axis.

c: Find the matrix describing a Lorentz transformation with velocity v along the x axis followed by a Lorentz transformation with velocity $\vec{v'}$ along the y axis. Does it matter in what order the transformations are carried out?

5: Griffiths 12.13
Sophie Zabar, a clairvoyant, cried out in pain at precisely the instant her twin brother, 500 km away, hit his thumb with a hammer. A skeptical scientist observed both events (brother's accident, Sophie's cry) from an airplane traveling at $\frac{12}{13}c$ to the right. Which event occurred first according to the scientist? How *much* earlier was it, in seconds? (Note that you are not told the horizontal position of the plane since it is not relevant.)

6: Griffiths 12.29
If a particle's kinetic energy is n times its rest energy, what is its speed?

7: Griffiths 12.33
A neutral pion of rest mass m and relativistic momentum $p = \frac{3}{4}mc$ decays into two photons. One of the photons is emitted in the same direction as the original pion's velocity and the other is emitted in the exact opposite direction. Find the relativistic energy of each photon.

8: Griffiths 12.34
In the distant past, most experiments in particle physics involved stationary targets (situation a): one particle (usually a proton or an electron) was accelerated to a high energy E, and allowed to collide with a target particle at rest. Far higher relative energies are obtainable (with the same accelerator) if you accelerate both particles to energy E and fire them at each other (situation b). You should assume that both particles have the same mass m. Classically, the energy $\bar{E}$ of one particle relative to the other is just 4E (why?). This is only a gain of a factor of 4. But relativistically the gain can be enormous. Show that:

$$\bar{E} = \frac{2E^2}{mc^2} - mc^2.$$

(a) (b)

Suppose that you use protons ($mc^2 = 1$ GeV) with $E = 30$ GeV. What $\bar{E}$ do you get? What multiple of E does this correspond to? Because of this relativistic enhancement, most modern elementary particle physics experiments use colliding beams instead of fixed targets.

9: Marion/Thornton 14.31
A neutral pion π^0 moving at speed $v = 0.98c$ decays in flight into two photons. If the two photons emerge on each side of the pion's direction with equal angles θ, find the angle θ and energies of the photons. The rest energy of a π^0 is 135 MeV.

10: Relativity - 1
Let's examine relativistic projectile motion in two dimensions.

a: Starting from $\frac{dp_x}{dt} = 0$, determine $\dot{x}$ in terms of $\dot{y}$, p_x and whatever else you need.

b: Starting from $\frac{dp_y}{dt} = -mg$, determine $\dot{y}$ in terms of $\dot{x}$, p_{y0} and whatever else you need.

c: Now combine the results to determine $\dot{x}(t)$. What is $\dot{x}(t)$ in the limit that $t \to \infty$? Explain. What is $\dot{x}(t)$ if $\beta << 1$? Does your result make sense? Explain.

d: Now get $\dot{y}(t)$. What is $\dot{y}(t)$ in the limit that $t \to \infty$? Explain. Similarly, what is $\dot{y}(t)$ if $\beta << 1$? Does this result make sense? Explain.