PHYS332 Homework - Relativistic Electricity and Magnetism

1 : Wangsness 29.20

a: Show that the quantities $\vec{E} \cdot \vec{B}$ and $\vec{E}^2 - c^2\vec{B}^2$ are invariants.

b: Evaluate these quantities for an infinite plane wave in a vacuum.

c: Show that the statement that $\vec{E}$ and $\vec{B}$ are perpendicular has absolute significance. Thus, if they are perpendicular for one observer then they will be perpendicular for all observers.

d: Show that a field that is purely magnetic in one frame cannot be transformed into one which is purely electric. Similarly, show that a field that is purely electric in one frame cannot be transformed into one which is purely magnetic.

Note that this is about the same as Griffiths 12.46 with a couple extra parts. I will provide two solutions since they are very different for what is part **a** here.

2 : Griffiths 12.47

An electromagnetic plane wave of angular frequency ω is traveling in the x direction through the vacuum. It is polarized in the y direction and the amplitude of the electric field is E_0.

a: Write down the electric and magnetic fields, $\vec{E}(x, y, z, t)$ and $\vec{B}(x, y, z, t)$.

b: This same wave is observed from an inertial system $\bar{S}$ moving in the x direction with speed v relative to the original system S. Find the electric and magnetic fields in $\bar{S}$, and express them in terms of the $\bar{S}$ coordinates: $\vec{\bar{E}}(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ and $\vec{\bar{B}}(\bar{x}, \bar{y}, \bar{z}, \bar{t})$.

c: What is the angular frequency $\bar{\omega}$ of the wave in $\bar{S}$? Interpret this result. What is the wavelength $\bar{\lambda}$ of the wave in $\bar{S}$? From $\bar{\omega}$ and $\bar{\lambda}$, determine the speed of the waves in $\bar{S}$. Is it what you expected?

d: What is the ratio of the intensity in $\bar{S}$ to the intensity in S? As a youth, Einstein wondered what an electromagnetic wave would look like if you could run along beside it at the speed of light. What can you tell him about the amplitude, frequency, and intensity of the wave, as v approaches c?

3 : Griffiths 12.64
In a certain inertial frame S, the electric field $\vec{E}$ and the magnetic field $\vec{B}$ are neither parallel nor perpendicular at a particular space-time point. Show that in a different inertial system $\bar{S}$, moving relative to S with velocity $\vec{v}$ given by

$$\frac{\vec{v}}{1+v^2/c^2} = \frac{\vec{E}\times\vec{B}}{B^2+E^2/c^2}$$

that the fields $\bar{\vec{E}}$ and $\bar{\vec{B}}$ are *parallel* at that point. Is there a frame in which the two are *perpendicular*?

4 : Wangsness 29.26
Find the Poynting vector $\vec{S}$ for the uniformly moving point charge. Show that the net power radiated by this charge is zero (since as we know the charge should only radiate if it's accelerating).

5 : Wangsness 29.29
An infinitely long ideal solenoid is at rest in the frame S' with its axis parallel to the y' axis. It has n' turns per unit length and carries a steady current I'. Find $\vec{E}$ and $\vec{B}$ inside and outside the solenoid for an observer in S for whom the solenoid is traveling with constant velocity $\vec{v} = v\hat{x}$. Sketch the lines of $\vec{E}$ and indicate the nature of the charge distribution which must be associated with such an electric field. Show that this charge distribution is in qualitative agreement with the Lorentz transformation of the "4-current" J^μ. Lastly, will an electric field be observed by someone who sees the solenoid moving with constant speed along y? Explain.

6 : Relativistic E&M - 1
Verify that $F'^{\mu\nu} = \Lambda^\mu_\lambda F^{\lambda\sigma}\Lambda^\nu_\sigma$ where

$$F = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{bmatrix} \text{ and } \Lambda = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In other words, complete the matrix multiplication to show that the transformed fields agree with what we determined in class.